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SHOCK-TESTING METHODS

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ABSTRACT

This memorandum discusses various problems and con-
ditions that occur in mechanical shock testing in the
areas of inadequate test specifications, shaping of shock
pulses, instrumentation, and shock-pulse interpretation.
Techniques which can be utilized to alleviate these problems
are also discussed.

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Introduction

Mechanical shock testing is a relatively new field of engineering where many unsolved problems exist. A mechanical shock is difficult even to define. In most textbooks a mechanical shock pulse is defined as a transient condition or impulse imposed upon a test item for a period of time which results in a change of momentum of the test item. For practically all cases, the change in momentum is brought about by a change in the velocity in magnitude and/or direction of the test item. The change in velocity is accomplished by subjecting the mass to a controlled variation in acceleration for the period of time involved.

Unfortunately, the above definition does not distinguish between the different effect a centrifuge test or a shock test of the same acceleration level has upon a test item. A better definition might be that a mechanical shock pulse is an external transient disturbance applied to a test item at some rate to which all the component parts of the test item cannot immediately respond. The transient disturbance can be either in the form of displacement, velocity, or acceleration. This means that because a transient is aperiodic (that is, it does not repeat itself regularly in the time interval observed), a test item undergoing a shock test is not in a steady-state condition during the shock; all the component parts of the test item are responding in different fashion. There are relative displacements and phase shifts between the responsive motions and the input motions. Therefore, all components within the test item might have different stresses which would not be present before and after the shock pulse of interest. After the shock, the steady-state condition of the component parts of the test item probably will be different than before the pulse, especially if little or no damping is present in the system. The reason is that these particles or components are all part of multi-degree, spring-mass systems, each of which responds differently to the transient.

This paper discusses problems occurring in shock testing from the time a test engineer receives a request for a shock test to the time the consultant receives the data he needs. The basic reason for conducting shock tests is .o determine the ability of the test item to withstand rapid applications of forces and stresses and still function properly either during or after the tests. The problem areas discussed in this memorandum are:

1. Nomenclature

2. Test specifications

3. Shock-machine requirements

4. Shock-pulse shaping

5. Fixture design

6. Shock-test instrumentation

7. Shock-pulse interpretation

LIST OF ILLUSTRATIONS (cont.)

The term, "shock tests," here refers to the type of test which is performed completely on a machine or facility and not to a free-fall drop where the item is not constrained to impact at a specified point. Tests in the second category are generally called "drop tests."

Nomenclature

Many of the terms used in the field of shock testing will be discussed in this memorandum. In performing a shock test on a test item, a basic problem is to record the input condition which is called the primary shock pulse. (See the enclosed circle in Figure 1.) The shock pulse is usually recorded in the coordinate system with acceleration as the ordinate, and time as the abscissa. The primary shock pulse is only a portion of the entire acceleration-time history that a machine carriage containing a test item will see during a shock test. The remainder of the acceleration-time history of concern will occur between the time the carriage starts from rest until it again comes to rest after the test.

Since the coordinates of a shock picture are time and acceleration, the units associated with these quantities will be discussed.

In most fields of engineering, time is usually measured in units of seconds. In shock testing, the duration of the shock is so short that time is better expressed in a smaller unit, such as milliseconds (ms). A millisecond is one one-thousandth of a second. Thus, if a shock test with a duration of 0.001 second was requested, the conversion to the smaller unit would appear as follows:

$$t[sec] = 0.001 \ sec$$

or

$$t[ms] = (0.001 \ sec)(1000 \ ms/sec)$$

from which

$$t[ms] = 1.0 \ ms.$$

Accelerations during shock must be in some unit of measurement. In other engineering fields, acceleration is generally measured in feet per second per second, or inches per second per second. In shock work, acceleration levels are of such high magnitude that the numbers associated with the usual units are cumbersome. It has been found convenient, therefore, to express shock acceleration in a larger unit, such as "gravities" or "gravitational unit," which has been contracted to "gees" or "gee," which is represented by the small letter "g." The magnitude of a ft/sec/sec unit is 1/32.2 of a g unit. The magnitude of the g unit selected permits ease of certain calculations. For example, if a shock test were requested where the peak acceleration was to be 32,200 ft/sec/sec, the conversion of acceleration to g units would be:

$$a[ft/sec/sec] = 32,200 \ ft/sec/sec,$$

or

$$a[g] = \frac{32,200 \ ft/sec/sec}{32.2 \ ft/sec/sec/g}$$

from which

$$a[g] = 1000 \ g.$$

Accelerations in this paper will most frequently be expressed in g units.

In connection with shock accelerations, it is necessary to define a new quantity, G, which is used in mechanical shock testing.
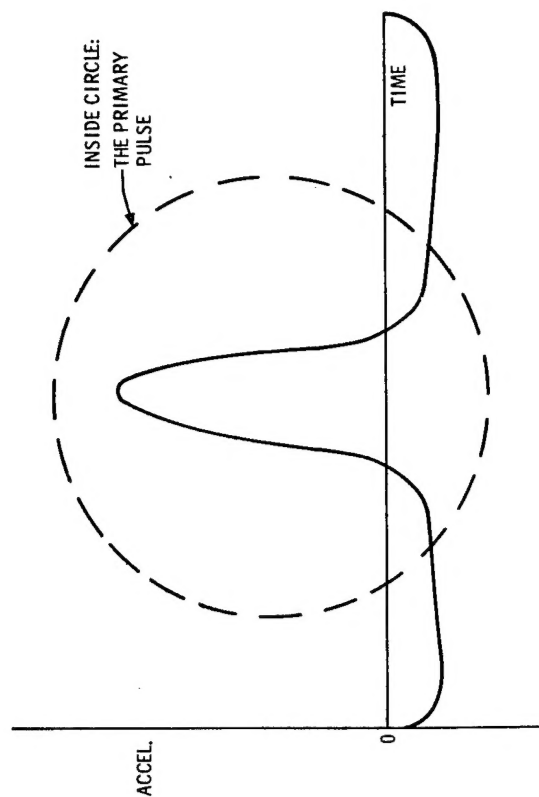
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Figure 1. Primary Pulse Portion of Acceleration-Time History

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From Newton's second law, we have the familiar

$$F = ma, \qquad (1)$$

By definition,

$$m = \frac{W}{g} .$$

Equation 1 can thus be written as

$$F = \frac{W}{g} a, \qquad (1a)$$

where

a = Any acceleration of mass, m (ft/sec/sec)

g = Acceleration of gravity (ft/sec/sec)

F = Any force causing mass, m, to accelerate (lbs)

W = Weight (lbs)

Equation (1a) then can be written as

$$\frac{F}{W} = \frac{a}{g} = G, \qquad (2)$$

where G is thus defined as the ratio of any acceleration to the acceleration due to gravity. (In shock testing, the acceleration would be any shock acceleration.) G is also the ratio of the necessary applied shock force to the carriage weight. G is, therefore, a pure number; however, the ratio is numerically equal to the shock acceleration measured in units of g.

Following is a description of the various parameters of the primary pulse:

Zero Acceleration Line (Zero Reference Line)

The zero acceleration line in a shock picture represents the condition of a shock machine carriage when it is at rest or moving at constant velocity and has a balanced force system on it.

Base Acceleration Line

The base acceleration line represents the amount of acceleration received by the test item before the application of the primary shock pulse. This acceleration may or may not be zero in magnitude, depending upon the type of shock machine generating the primary pulse. The acceleration level of the carriage after the primary pulse in most cases will be very nearly the same as before the primary pulse. This base acceleration level must be considered in some tests, for some stresses created within a test item are dependent upon it. But in every case, when a vertical machine is used, the base acceleration line on the shock picture will be located 1 g from the zero acceleration line in the direction of the acceleration due to gravity, if gravity would be shown in the shock picture.

Maximum Faired Acceleration

The maximum faired acceleration is defined as the maximum amplitude of a smooth ideal pulse measured from the zero reference line as shown in Figure 2. The actual maximum acceleration of the carriage is represented by the distance from the zero acceleration line to the maximum amplitude of the pulse. The change in acceleration which causes the shock motion within the test item is represented by the distance from the base acceleration magnitude to the peak amplitude as shown in Figure 2. Maximum faired amplitude is sometimes synonymous with peak amplitude. The term, "faired amplitude," means the smooth curve (drawn through higher frequencies of a pulse) which is used to correlate the amplitude of a recorded pulse to the amplitude of the requested pulse. More details of pulse fairing will be discussed in the section, Shock Pulse Interpretation.

Duration

The shock duration is actually the interval of time during which the acceleration level of the primary pulse is significantly greater than the base acceleration level. However, a shock pulse has a duration at any amplitude level during which the amplitude is greater. Since the maximum faired amplitude of the shock is measured from the zero reference line, the maximum shock duration for the pulse is sometimes referred to the zero reference line.

Since duration can be measured at any amplitude, it has been found convenient at Sandia arbitrarily to measure the duration at the 10-percent faired amplitude level to gain reading accuracy of some pulses. Figure 3 illustrates the duration of the shock pulse measured at the zero reference line, the 10-percent faired amplitude line, and the symbols that will be used to identify them throughout this memorandum.

Jerk

The jerk of a shock pulse is the time rate change of acceleration (slope of an acceleration-time curve). It is positive whenever the acceleration is increasing, and negative when the acceleration is decreasing. The jerk is one of the main conditions which causes a test item to respond under shock in the way it does.

Rise Time and Fall Time

Rise time is the time interval when the jerk of a smooth acceleration pulse is positive. Rise time can never be longer than the interval required to have the smooth shock acceleration increase from the base acceleration level to the peak amplitude point. Any interval of time less than the maximum can be selected, however, where it is convenient to consider the rise time. At Sandia it is convenient to consider the rise time as measured between the 10- to 90-percent faired amplitude levels as the shock pulse progresses.

Similarly, fall time is the interval of time when the jerk of the pulse is negative. Fall time can be compatible with the rise time, fall time is measured as the time interval between the 90- to 10-percent faired amplitude level as the shock pulse progresses (see Figure 4).

Velocity Change

Velocity change is the area beneath the acceleration-time history, during the primary pulse. It represents a change which occurs in the velocity of the shock machine carriage from the beginning of the pulse until the end of the pulse.
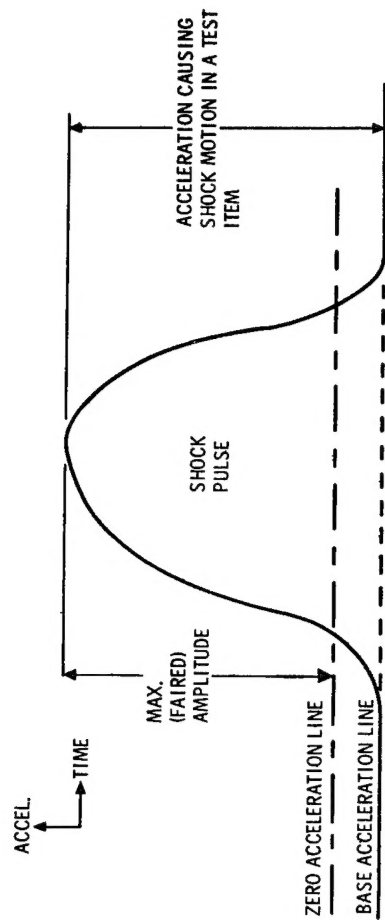
Figure 2. Shock Pulse Amplitudes (General Case)

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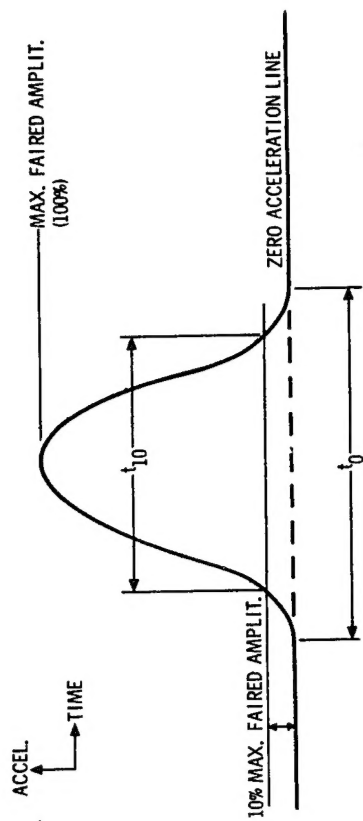
Figure 3. Shock Pulse Durations

MAX. FAIRED AMPLIT. (100%)

ZERO ACCELERATION LINE

t_{10}

t_0

10% MAX. FAIRED AMPLIT.

ACCEL

TIME

Figure 4. Shock Pulse Rise Time and Fall Time

100% MAX. FAIRED AMPLIT.

ZERO ACCEL. LINE

FALL TIME t_f

RISE TIME t_r

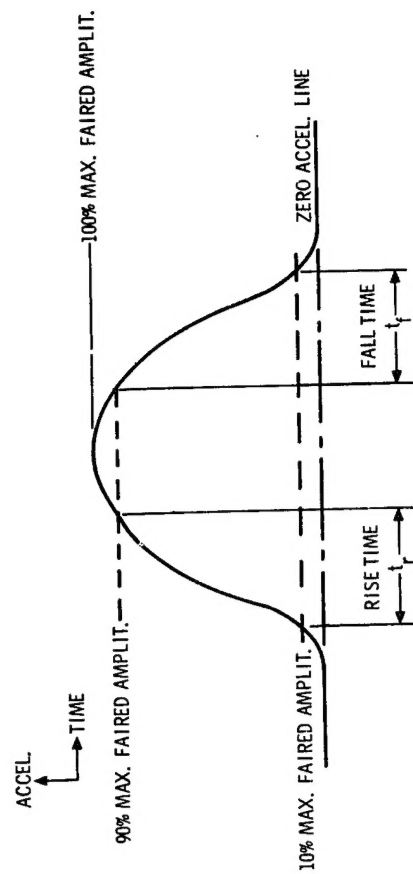90% MAX. FAIRED AMPLIT.

10% MAX. FAIRED AMPLIT.

ACCEL

TIME

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In the shock-testing field, velocity change is measured in feet per second, or sometimes in a new unit, "gravity-seconds," which by common usage has been contracted to "gee-seconds" which is written as "g-sec." For example, if the velocity change from a shock pulse is 64.4 ft/sec, conversion into the other set of units would be

$$\Delta V \, [\text{ft/sec}] = 64.4 \text{ ft/sec},$$

or

$$\Delta V \, [\text{g-sec}] = \frac{64.4 \text{ ft/sec}}{32.2 \text{ ft/sec/sec/g}}$$

from which

$$\Delta V \, [\text{g-sec}] = 2.0 \text{ g-sec.}$$

Figure 5 graphically shows the area to be considered for velocity change determination of the carriage.

Pulse Shape

The primary pulse shape describes the portion of the acceleration-time history that is of most interest. Figure 6 shows some standardized pulse shapes which can be approximated on facilities at Sandia Corporation. A simple shock-pulse shape can be either symmetrical or nonsymmetrical; a more complex pulse will contain a combination of two or more of the simple symmetrical shapes. The standardized pulse shapes are shown in nondimensional form, which means that time has been divided by duration at the zero reference line, and acceleration has been divided by the maximum faired amplitude.

Test Specifications

A major problem area lies in the test specifications which the test engineer receives from the consultant. To conduct a properly designed shock test, it is desirable that the test engineer know information which is contained in 13 points. The first four points deal with the primary shock itself in terms of the kind of shock test to which the test item is to be subjected. The last nine points deal with details of the shock-test setup.

Maximum Acceleration (Maximum Faired Acceleration, and Peak Acceleration)

The magnitude of the maximum acceleration, or accelerations if several different peak levels are desired, for a shock test is generally governed by the consultant's desire to see certain loads which might be expected to occur on a test item during its use and to know if the item functioned properly. The maximum acceleration level is sometimes referred to as the nominal value of the amplitude of the shock. The peak and maximum faired acceleration mean the same as maximum amplitude in a specification, but in a recorded pulse they are different. (Refer to Figure 23 under Pulse Interpretation.)

An absolute tolerance of at least ±15 percent should be allowed on the desired peak accelerations to be generated, because it is difficult to predict accurately what the peak should be under the present knowledge of the state of the art. The over-all tolerance, ±15 percent, includes instrumentation and machine setup where machine setup is as accurate as possible.

Duration

Quite often a duration is specified but the amplitude level of the duration is not mentioned. Most consultants assume that duration is to be measured at the zero reference line. This assumption creates a problem on some pulse shapes where the pulse start and stop are not sharply defined, e.g., a parabolic cusp. For some shock tests, however, the duration of a pulse might not be important. The degree of importance can only be determined when the consultant compares it to the test item natural period. When a shock duration is specified, the amplitude level at which it is to be measured must also be specified. If no amplitude level is specified, then the duration will arbitrarily be measured at the 10-percent faired amplitude level in accordance with SC-4452B(M).
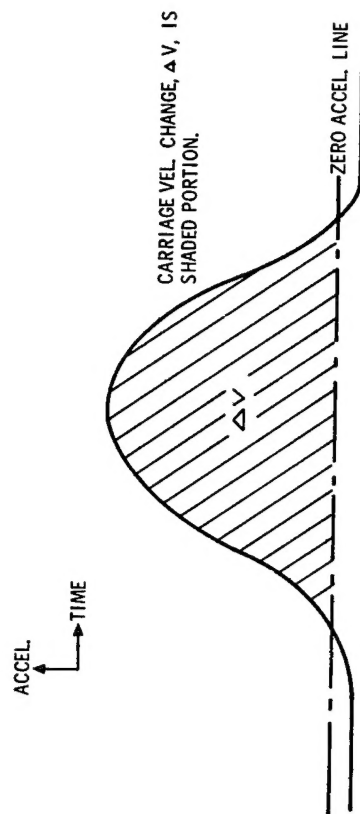
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CARRIAGE VEL CHANGE, Δ V, IS SHADED PORTION.

ZERO ACCEL LINE

ACCEL

→ TIME

ΔV

Figure 5. Velocity Change During Shock Pulse of Carriage

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Since durations are specified in shock-test work, a tolerance must also be allowed, because the duration is quite difficult to predict and to keep constant in most shock tests. The absolute tolerance for durations of generated pulses should never be less than ±15 percent; for short durations, around 1 millisecond or less, the tolerance should be at least ±30 percent. This tolerance includes instrumentation error.

Pulse Shape

Each different type of shock-pulse shape occurring in a shock test can produce a different response from the test item. Sometimes a compromise will have to be made between the pulse shape that the consultant desires in a test and that which can be produced on a particular shock machine. This compromise might involve amplitude, duration, and pulse shape, simultaneously.

The Appendix shows shock spectra (Figures A1 to A8) of undamped, single-degree, linear spring-mass systems subjected to the different standardized shock pulses. The value for the duration of the pulse is determined from the zero reference line. Points on each "primary spectrum" represent the ratio of the maximum responsive acceleration of the mass to the input acceleration occurring in the same direction. Points on each "residual spectrum" represent the ratio of the maximum responsive acceleration to the input acceleration that occurs in free vibration of the mass after the pulse is over. Examination of these various shock spectra indicates that one need not restrict oneself to specifying only half-sine, square, or saw-tooth pulse shapes. A predictable acceleration response can be made for any shock pulse in Figure 6.

If the pulse shape does not fall into a category shown in Figure 6, then the area or velocity change of the pulse needs to be specified. (The velocity change of a standard pulse is established and can be calculated from the nomograph shown in Figure A9 in the Appendix.) The velocity change of a pulse is the measure of the energy required of a machine to generate this shock when the carriage weight and its velocity at the pulse start are known.

Whenever the area of requested pulse is specified, a tolerance must be allowed. An absolute tolerance of ±25 percent on any requested pulse-shape area can be met by most shock test laboratories.

Rise Time

Since rise time is a measure of the jerk which affects maximum response of the test item, the rise time should be specified with a tolerance, because pulse shapes can only be approximated. The rise time must always be specified between two amplitude levels. If no levels are specified, rise time is assumed to be measured from 10 to 90 percent of the faired amplitude level, in accordance with SC-4452B(M). This amplitude range is arbitrary. Some error is involved in selecting the rise time between the 10- to 90-percent amplitude levels for most pulse shapes, but the error is not as much as would occur if rise time were taken from the 0- to 100-percent amplitude levels. (The nomograph shown in Figure A9 can be used to determine the rise time for the various shapes shown in Figure 6. For square pulses an arbitrary rise time must be specified.) The rise time of a generated pulse should not be held to an absolute tolerance less than ±15 percent.

Tolerances mentioned in the first four items above do not represent an allowable range of a recorded pulse to deviate from a requested pulse, but in reality represent total allowable deviations of a generated pulse from the requested pulse.

Definition of the Test Item Axes

The three orthogonal or perpendicular principal axes of the test item must be shown or described adequately so that the test item can be correctly oriented in a shock test. The axes can be identified by letters, such as X, Y, and Z, or by designations such as longitudinal, vertical, or lateral. (Figure 7 shows an example of axis designations.) The terms, axes and planes, are sometimes used interchangeably, when the shock application is specified (e.g., the shock is to be applied in the X-axis or through the X-plane).
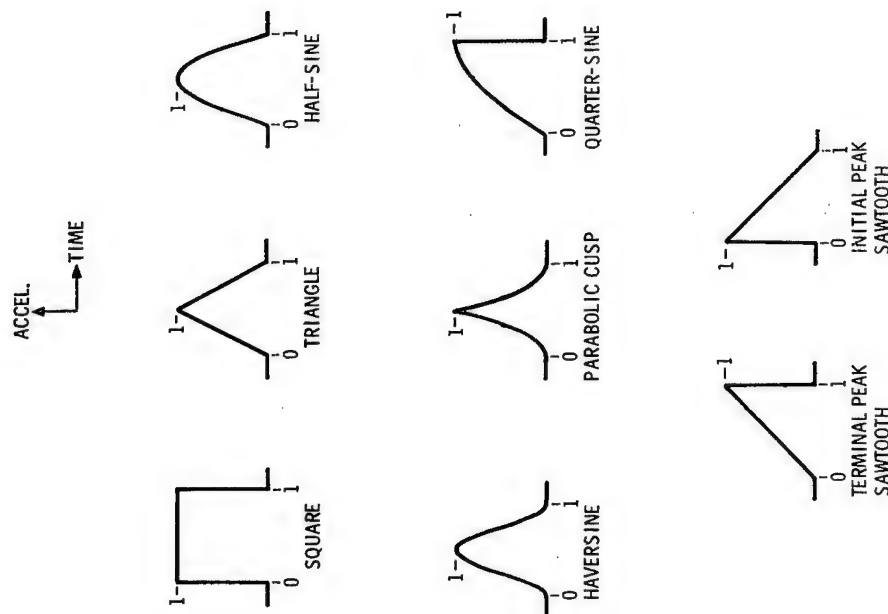
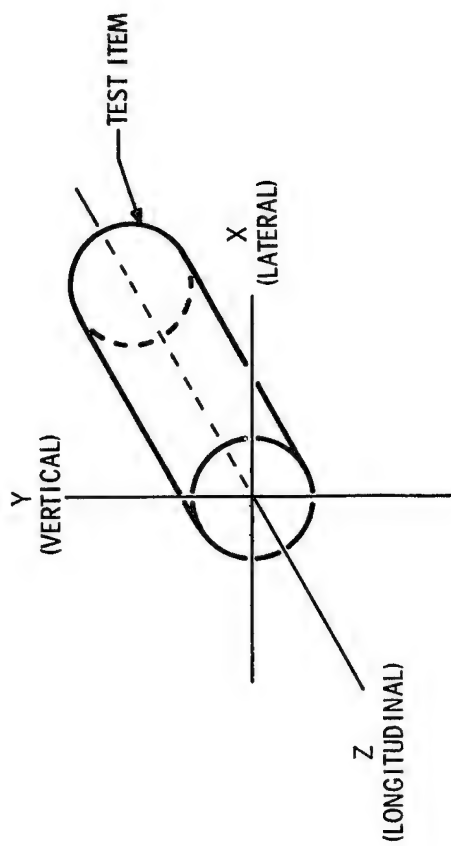

Figure 6. Standard Shock Pulse Shapes

Figure 7. Axis Designation

Describing a shock application through a plane might be satisfactory if it were not for the fact that such a description could be ambiguous. When the "X-plane" is mentioned, the question arises as to whether the shock will be applied perpendicular to the Y-Z plane in the X axis, or whether it will be applied in a plane formed by the X-Z axes or X-Y axes. In the latter two cases the shock could be in any direction in either plane.

It is best to specify a definite axis when shocking a test item.

Directions of Applied Acceleration

In vibration tests, acceleration directions are cyclic in nature, repeating themselves in certain periods of time. In shock tests the acceleration direction during a primary shock is either in one or the other direction of a certain axis. Because of this condition, it is necessary that a specific direction be requested for application of the acceleration. It is customary to label directions parallel to the coordinate axes of a test item; for instance, +X, -X, +Y, or X1, X2, Y1, etc. Figure 8 shows three suitable methods of specifying applied shock direction for shock tests. (A right-hand coordinate system has been used.) Other methods of specifying directions may be satisfactory, provided that they are not confusing.

In setting up a specific direction for the shock acceleration (the applied force) in a shock test, personnel in the shock laboratory need not be concerned with the function or response of a test item during the shock. The test item can be considered to be a little black box (having unknown internal parts) which is to receive an input shock of certain conditions in a certain direction. Since the person requesting the shock test decides the direction in which the shock is to be applied, it is very important that he be aware of the direction which relative motion of internal parts will take during a primary shock. This particular problem causes much confusion between those who request and those who conduct the shock tests.

An example is in order here. Assume that the switch armature in the test item shown in Figure 9 is to close against contact B during the primary shock. The problem is to specify the direction of the external force, F (and, hence, the acceleration), to the test item. Assume that the test item is initially at rest, and then the primary shock is applied. From the figure, one can see that during the primary shock, the applied force, F, must be to the left, causing contact B to be forced toward the switch armature when all motion is referred to a stationary reference frame, namely, the earth. The inertia of the switch armature will cause it to remain at rest until contact B strikes it and forces it also to move to the left. If the support was quite strong, the switch armature would have moved to the left before contact B struck it. In any case, contact A would not be touched before contact B if the force, F, was to the left.

If the test item is moving at constant velocity, V, to the right, and then the primary shock is applied to the left, the inertia of the armature will cause it again to strike contact B. Carrying this example a little farther will show that, to have the armature hit contact B, the applied force must be in the same direction (to the left), regardless of the direction or magnitude of velocity of the carriage at the start of the pulse. It might, therefore, be noted that the applied acceleration during the primary pulse does not have to be in the same direction as the test item velocity.

When one uses the inertial force concept in the problem shown, one is essentially assuming that the test item is a fixed reference frame and some force is acting on the armature to the right, causing the switch armature to move to the right. If this method is used, the instructions should clearly state the direction and type of force that is assumed - either applied or inertial. In the shock laboratory, the little black box technique (applied force technique) is preferred in setting up a shock test.

Number of Shocks Applied in Each Direction

The number and type of shocks that a test item receives in each direction in a shock test are governed by what the item might experience in the field, between the time of its manufacture to its ultimate delivery. Sometimes the number and type of shocks are deliberately increased to overtest the unit. Therefore, frequently the test specification will require several shocks (usually three) to be applied in each one of the six orthogonal, perpendicular directions. If a unit happens to be marginal, the first shock in a given direction could weaken the structure, and a second or third shock could cause a failure.
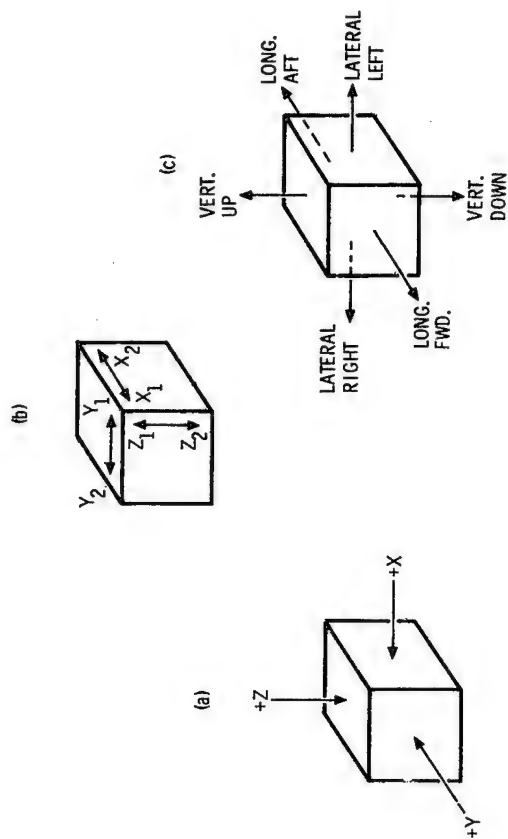
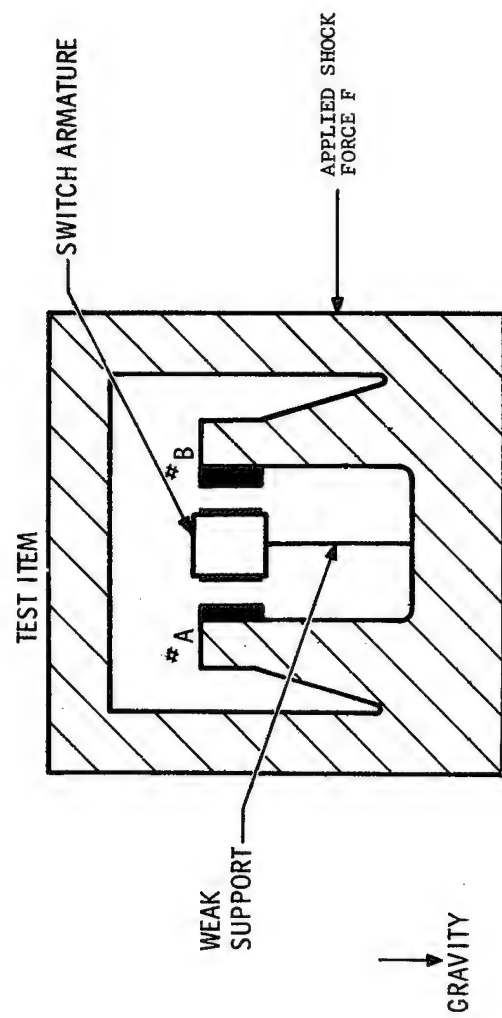Figure 8.   Three Methods of Specifying
Applied Shock Direction

Figure 9. Example of Shock Direction Determination

Sequence of Shocks

If a test item is to be shocked in more than one direction, the sequence of directions for the shocks should be mentioned. In some cases the sequence is not important, but in others it could be, especially if the test item is more sensitive in one direction than in another; e.g., a weapon is launched and impacts some distance away, and then later explodes. Here, two different shocks will occur in opposite directions. In simulating both shocks in the laboratory, performing the last shock first, and the first shock next, might be detrimental to the unit, while reversing the shock sequence would not be.

Limits on All Accelerations Other Than the Primary Shock Pulse

The primary shock mentioned in the test request is not the only input acceleration experienced by a test item from the start to the conclusion of the shock test. Since the carriage on which the test item is mounted starts from rest and ends at rest, (if the test item is to be recovered), accelerations have to occur in more than one direction. At Sandia, secondary acceleration levels during a test generally are less than 20 percent of the requested acceleration peak of the primary shock. If secondary accelerations are ignored, the test item might still be responding from the preliminary accelerations at the time the primary shock pulse is applied, or else might respond adversely to the secondary shock pulses after the primary shock is over.

Test Item Mounting Requirements

If not otherwise stated, the test item should always be mounted to the shock-test fixture in the same manner in which it would be mounted to the next assembly in its normal application. If the normal mounting cannot be accomplished, then a suitable substitute method must be determined. Substitute mounting techniques are probably used in the early developmental test of a new test component. The substitution might consist merely of strapping the unit to the carriage instead of fastening it to a fixture by using bolts. The reason the mounting technique is important is that changes in the technique from one test to another can cause different responses of the test item to the input shock. Refer now to Figure 10. Considering only the small component it can be shown that if the shock duration is greater than ten times the natural period of the subassembly, then either mounting is adequate. If the duration is less than ten times the subassembly natural period, then the right-hand mounting configuration is preferred.

Considering the entire subassembly, one can see that the point of application of the shock could be important. In the left-hand configuration the shell is in tension. In the right-hand figure the shell is in compression and possibly is buckling. The stresses in the shell undoubtedly would be different. The shock motion of the upper weight could also be considerably different.

The accelerometer used to monitor the input shocks to a particular item in all cases should be mounted close to the mounting points of the item. The location of accelerometer No. 1 in the left-hand part of Figure 10 generally would be unsuitable.

If the test item has long external electrical cables which are massive in size, the method by which these cables are looped or fastened to the carriage, so that the shock test will not destroy the cabling, must be considered. Similarly, possible cable whip feeding back into the test item must be considered after the primary shock.

Mechanical Shock Tests Conducted at Extreme Temperature

When mechanical shock tests are to be conducted while the test item is at an extreme temperature, the test engineer must know certain requirements. Normally, the test item is preconditioned at the extreme temperature, transferred and placed on the shock machine at room temperature, and then shocked. The maximum permissible time from the removal of the test item from the extreme temperature chamber to the completion of the last shock should be indicated in the test specifications. The particular location on the test item where temperature is to be controlled should be known. If the electrical connectors and contacts have to be protected from frost condensation, this should also be indicated. If the test item is to be insulated during the time it is in shock, then a special test fixture might have to be designed.
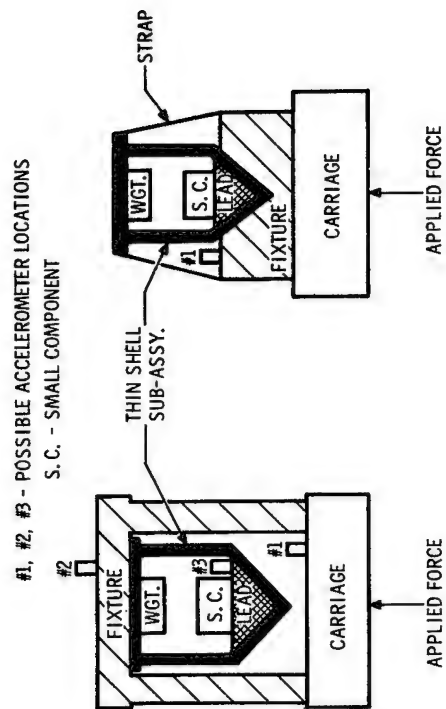
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#1, #2, #3 - POSSIBLE ACCELEROMETER LOCATIONS
S. C. - SMALL COMPONENT

Figure 10. Two Different Mounting Techniques

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Frequency Response of Monitoring Equipment

To conduct a satisfactory instrumented shock test, the engineer must know what range of frequencies should be monitored by the input pickup circuitry. If this is not known, results of the test might be compromised by the type of instrumentation selected and the method of reducing data.

Test Item Functional and Structural Response

If the response of a test item to an input shock is to be monitored, the location, orientation, and maximum allowable weight of pickups must be known. Whether the monitoring is to be done during, before, or after the test, must also be known. If monitoring is to be done during the shock test, instrumentation should be set up to monitor the functional or structural response to the same time base reference as the input shock. The effect of violent cable movement during the test upon electrical measurements should be considered.

If the above points are considered in a shock-test specification, better shock tests will be accomplished.

Shock Machine Requirements

At Sandia, mechanical shock-test machines are classified as "impulse" or "impact" shock machines. The term, "impulse," refers to a machine that increases the test item velocity as a result of the shock; the term, "impact," refers to a machine that either decreases the velocity or changes the direction of the velocity of the test item during impact. To select either machine for a requested shock, studies should be made of the kinematic relationships between the acceleration-, velocity-, and displacement-time pulses that the shock machine carriage will have to experience during the shock.

The acceleration-, velocity-, and displacement-time relationships of the shock pulse are important. The acceleration is important, for during the shock pulse it determines the forces that will be appearing throughout the shock machine. The velocity change (area under the acceleration-time curve) is important because it is either the sum of the impact and rebound velocities of the carriage, or the maximum velocity of the thrust column or piston. In turn, these velocities are a measure of the amount of energy necessary to generate the requested shock pulse when the carriage weight is known. The displacement occurring during a shock pulse is important for, along with the acceleration, it determines the necessary dynamic spring constant of the shock spring. The maximum displacement determines the minimum stroke length on impulse machines and the pad thickness on impact machines.

The relationships between acceleration-time, velocity-time, and displacement-time curves for any given theoretical pulse can be easily determined by the use of elementary calculus. The only parameters that have to be known are the velocity and the acceleration of the carriage at the start of the primary shock pulse. The slope of the displacement-time curve at any point represents velocity, while the slope of the velocity-time curve at any point represents acceleration at that point. The slope of the acceleration-time curve at any point is called the jerk. However, the slope of any curve at a given point cannot be determined accurately unless the equation of the particular curve is known, and, since acceleration-time relationships usually are monitored, other relationships should be used. From calculus, the area under an acceleration-time curve represents velocity, while the area under velocity-time curve represents displacement. So, by integration of the acceleration-time curve in some manner, all the necessary kinematic relationships of the shock test can be obtained.

A few detailed examples solved mathematically in the Appendix illustrate the various kinematic relationships between the carriage acceleration, velocity, and displacement for three pulse shapes, half-sine, square, and haversine, when generated on different facilities. The resulting curves of the analyses are shown in Figures 11, 12, 13, and 14 of the text. For two examples, assumptions are made that the pulse duration is measured at the zero reference line and that any carriage acceleration before and after each pulse can be neglected. In the third example, initial carriage acceleration is considered for a haversine pulse of low amplitude, long duration.
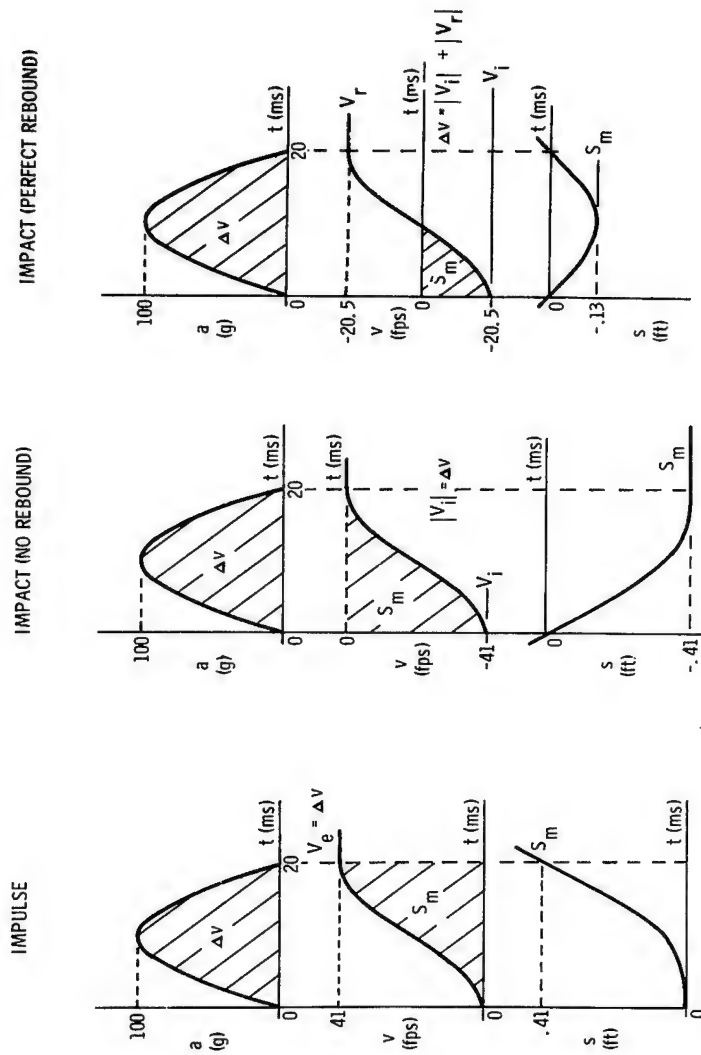
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Figure 11. Acceleration-, Velocity-, Displacement-Time Curves For 100 g, 20 ms Half-Sine Pulse Produced On Various Machines
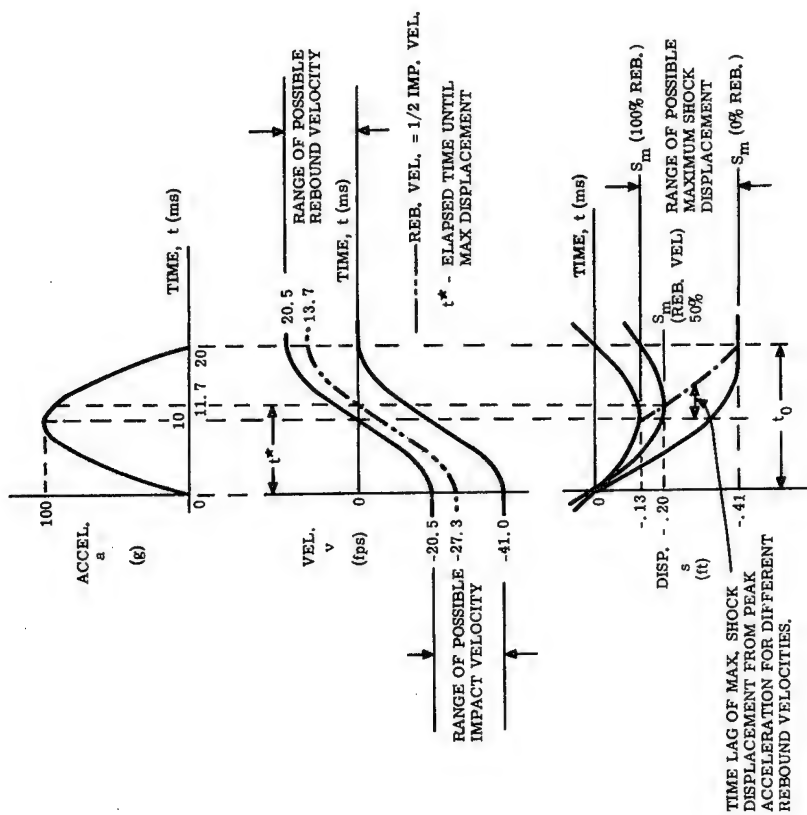
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Figure 12. Effect of Rebound Upon Velocity-, Acceleration-, Displacement-Time Curves for 100 g, 20 ms Half-Sine Pulse on Impact Machine
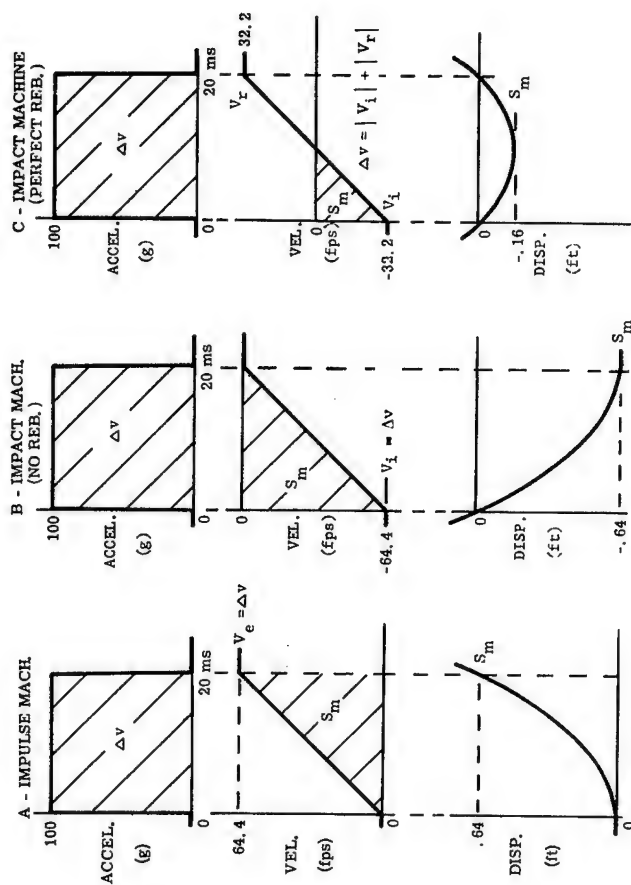
24

Figure 13. Acceleration-, Velocity-, Displacement-Time Curves
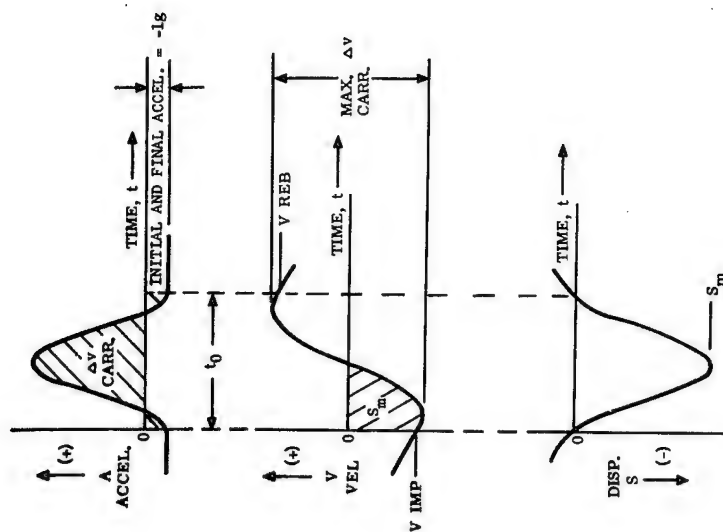for 100 g, 20 ms Square Pulse for Two Type Machines

In Figures 11a and 11b, the magnitude of maximum displacement, s_m, is the same, and the velocity either at the beginning or the end of the pulse equals the velocity change of the shock pulse. It can be shown that the same maximum shock displacement is required for any symmetrical pulse generated where the initial acceleration is zero, and where the velocity change and the duration at the zero line are the same. Figure 11c shows the curves for the shock pulse generated on a machine capable of producing perfect rebound. The velocity and displacement-time curves of Figures 11b and 11c prove mechanically that a half-sine pulse (in fact, any pulse shape) can be generated on an impact table where any rebound from 0 to 100 percent occurs. For a 100-percent rebound machine the energy requirements for a certain pulse would be one-fourth those of the same machine having no rebound.

Figure 12 shows velocity and displacement-time curves for a half-sine pulse generated on a machine where different rebounds occur. The dash-dot line in the displacement-time graph shows the time lag of maximum displacement from peak acceleration for different percentages of rebound velocities.

In Figure 13 the acceleration, velocity, and displacement-time curves for the square pulse shape follow a general pattern similar to the half-sine pulse shape of Figure 11.

In Figure 14, the carriage has an initial acceleration before the primary shock. For this case, the sum of the carriage velocities at impact and rebound is not equal to the carriage maximum velocity change, as it was in the previous examples.

Kinematic curves of acceleration-velocity-displacement-time pulses of the carriage can be drawn for any pulse shape, no matter how complicated, by using other techniques of integration.

Instead of calculating the various parameters of interest (such as maximum shock displacement and velocity change) which were shown in Figures 11, 12, 13 and 14, the mechanical shock nomograph shown in the Appendix (Figure A-9) can be utilized to determine the values of the parameters. The calculated values of half-sine and square pulse examples above can be compared to values obtained from the nomograph.

Shock-Pulse Shaping

To select either an impulse or impact shock machine to produce a requested shock input, the various dynamic conditions controlling the important shock parameters must be known; namely, the amplitude, the duration, and the pulse shape. Machines at Sandia which are capable of producing an impact-type shock are free-fall and accelerated-fall machines, inclined ramps, rocket sleds, and actuators used as velocity generators. The impulse shocks are produced on machines accelerated by shock cords, by gas expansion, or by rocket sleds. Producing the input shock on impulse or impact machines does not make any significant difference for a correct mounting configuration of the test item; the applied force to the item during the primary shock can always be put in the same direction in either case. (Refer to Figure 8, assuming the condition of a horizontal machine. In this figure the velocity of the test item at the start of the pulse can either be zero for impulse shock, or directed to the right for an impact shock.)

Conditions affecting the shock parameters as they would be generated on an impact machine on an impulse machine are examined first because shock-pulse shaping is more versatile on this type of facility than on an impulse machine. All pulse shapes shown in Figure 6 can be generated on impact machines.

Impact Machine

First considerations of a pulse generated on an impact machine would be the weight and size of the carriage in use and the velocity of the carriage at the start of the impact. The carriage moving at some velocity at the start of the pulse implies that some means must be employed to generate this velocity. If only gravity is used to activate a free-fall carriage, then the drop height is considered. Otherwise, the force required initially to get the carriage up to speed must be considered.

Figure 14. Acceleration-, Velocity-, Displacement-Time Curves for Haversine Shock Pulse Produced on Free-Fall Machine

When only gravity is utilized, and friction can be neglected on a vertical free-fall machine, the drop height required to get a desired impact velocity can be found by the energy equation

$$Wh = \frac{1}{2} \left(\frac{W}{g} \right) v_i^2 \tag{3}$$

or

$$v_i = \sqrt{2gh} \tag{4}$$

where
    W = Total carriage weight (lbs)

    h = Drop height (ft)

    g = Magnitude of acceleration due to gravity (ft/sec/sec)

    v_i = Carriage velocity at impact (ft/sec).

If a single pendulum-type machine is used, the maximum impact velocity can be found from the energy equation

$$WL(1 - \cos \theta) = \frac{1}{2} \left(\frac{W}{g} \right) v_i^2$$

or

$$v_i = \sqrt{2gL(1 - \cos \theta)} \tag{5}$$

where
    L = Length of suspension cable (ft)

    θ = Release angle from vertical axis (deg).

By using a double pendulum it is possible to obtain

$$v_i = \sqrt{3gh} \ . \tag{6}$$

Figure 15 is a sketch of a machine, where Equation 6 had to be used. The proof of Equation 6 is shown in the Appendix.

If a vertical free-fall carriage is accelerated downward by linear elastic cords, then the impact velocity can be approximately determined from the following energy equation

$$\left(W + \frac{P_r + P_i}{2} \right) h = \frac{1}{2} \left(\frac{W}{g} \right) v_i^2$$

from which

$$v_i = \sqrt{2gh \left(1 + \frac{P_r + P_i}{2W} \right)} \tag{7}$$

where
    P_r = Shock cord dynamic preload at release (lb)

    P_i = Shock cord dynamic preload at impact (lb).

(The kinetic energy of the elastic cords has been neglected.)

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HOIST CABLES

TORQUE

SUPPORT

SUSPENSION RODS

GUIDE ROD

IMPACT PADS

CARRIAGES

TEST SPECIMEN

DUMMY WEIGHT

CARRIAGE VELOCITY

CARRIAGE VELOCITY
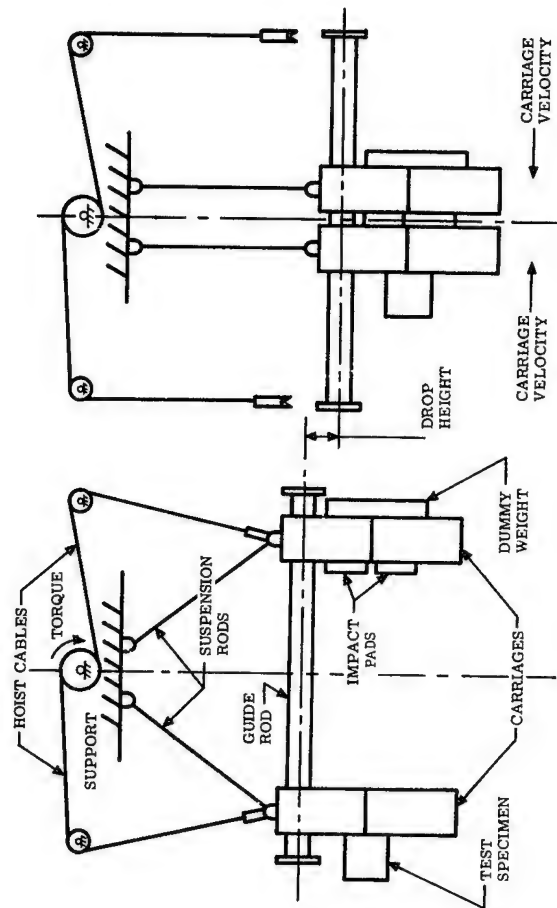
DROP HEIGHT

a. Cocked Position

b. Impact Position

Figure 15. Double Pendulum Shock Machine

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Drop height in all these examples is the vertical height through which the carriage falls from release to the instant at which the shock spring is touched.

During impact (at which time the carriage velocity is rapidly changing), forces of high magnitude will occur between the carriage and the machine base. By controlling the magnitude of the force throughout the shock deflection in any certain way, a characteristic shock-pulse shape will develop. The load-deflection control constitutes the makeup of the so-called "shock spring."

Creating a desired pulse shape for impact machines can be quite a challenge and frequently cannot be done. Until more impact materials are evaluated, an undesirable shape of a generated pulse will sometimes result. The cause of the shape is the dynamic spring constant of the shock spring. The design of a shock spring for a particular pulse shape is difficult, because the dynamic spring constant of the impact materials seldom is equal to the static spring constant which generally can be easily calculated or measured. Such shock spring materials are thus considered strain-rate sensitive. It was shown that a half-sine pulse could be produced using a spring having various amounts of resiliency. If the spring-mounting surface is assumed to be rigid, the more resilient materials cause higher rebound. Low rebound means that the spring during the impact is acting as a visco-elastic material, which means that the carriage rebounds faster than the spring can recover.

Several materials in a suitable shape can be used to act as a spring during the shock. All the mechanical properties and dimensions which are needed for a good spring design have to be considered. When using an impact pad or cushion the spring during compression will have kinetic energy of its own, perpendicular to the shock axis, and the dimensions of a pad during dynamic compression might not be the same as they would be during a static compression. In this case, a bearing surface or friction between pads and anvil and carriage bottom will have some effect on the shock pulse.

Generating a Half-Sine Pulse

What is required to generate a half-sine shock pulse and how its amplitude and duration can be changed will now be discussed. This pulse shape is selected first because of the ease in showing theory. In actual practice, however, a half-sine pulse is never achieved. It is only approximated because the jerk of a shock pulse can never have a discontinuity which occurs at the base acceleration level for a half-sine pulse. The approximation or closeness to a half-sine, however, can be quite good for shocks having relatively low amplitude and long duration.

The most common design of shock springs used to generate a half-sine pulse are steel leaf springs or rubber pads. Figure 16 shows what the dynamic load-deflection curves must look like when materials having different resiliencies are selected to create a half-sine pulse. The maximum force, F_m, obtained from Figure 16 can be correlated to the maximum shock amplitude, G_m, by knowing the carriage weight, W, and using Equation 2.

To find out what conditions affect the amplitude of this type of pulse, a certain spring material and shape is assumed so that the dynamic load-deflection curve is essentially linear up to the maximum load. (This linearity will occur with little error if rebound velocity is at least 50 percent of impact velocity.) The kinetic energy loss of the carriage assembly can be equated to the strain energy gained by the spring during the shock.

Equating energies

$$\frac{1}{2}\left(\frac{W}{g}\right)V_i^2 = \frac{1}{2}\left(F_m\right)S_m \ .$$

(8)

By making the substitution

$$F_m = W\,G_m \quad \text{and} \quad F_m = k\,S_m \ ,$$

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PLASTIC MAT'L

VISCO-ELASTIC MAT'L

ELASTIC MAT'L

F_m

F LOAD

DEFL, S

NOTE: Distance between dots occurs during time intervals = 1/10 pulse duration, t_o.

S_m (100% REB. VEL.)

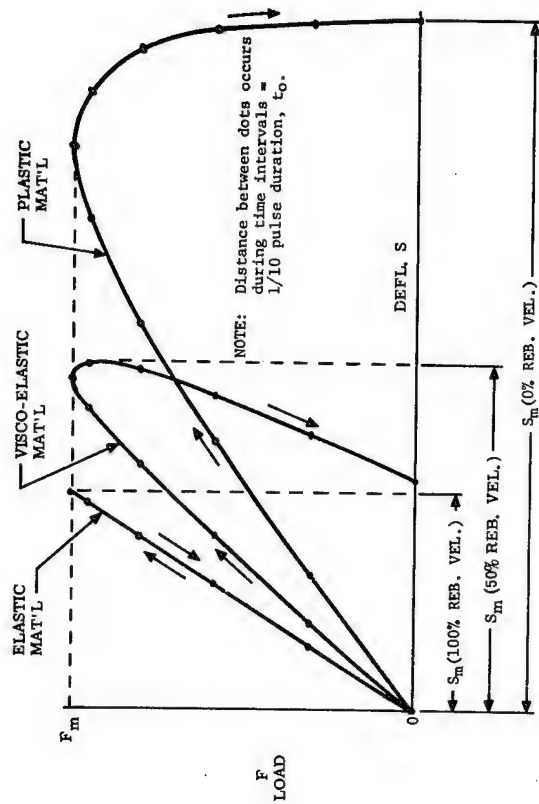S_m (50% REB. VEL.)

S_m (0% REB. VEL.)

Figure 16. Dynamic Load-Deflection Curves of Various Materials for a Shock Spring to Generate a Half-Sine Pulse

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Equation 8 becomes

$$G_m = V_1 \sqrt{\frac{k}{gW}}$$ (9)

where   k = Dynamic spring constant

(Other parameters have already been defined.)

Equation 8 is useful whenever a rebound velocity of at least 50 percent occurs. If a free-fall drop table is used, Equation 9 can be written (using Equation 4) as

$$G_m = \sqrt{\frac{2hk}{W}}$$ (10)

Similarly, for a free-fall drop table, Equation 8 can be used to show that

$$G_m = \frac{2h}{S_m}$$ (11)

where $h >> S_m$.

From Equations 9 and 10, once the spring constant, k, and carriage weight, W, have been determined, the maximum shock amplitude, G, depends only upon impact velocity, V_1 (or drop height, h, in a vertical machine).

The duration of a half-sine pulse is found by realizing that the carriage in contact with a linear shock spring constitutes a single-degree linear spring-mass system during the time of the shock. Therefore, the equation for shock duration is

$$t_o = \pi \sqrt{\frac{W}{gk_e}}$$ (12)

where k_e = spring constant for perfect elastic spring.

Equation 12 is identical to the equation of the half-period of a linear spring mass system that would be in continuous vibration.

From Equation 12 the duration can be said to be dependent upon the shock-spring material and the carriage weight. The impact velocity does not affect duration once k_e has been determined. Therefore, to generate a specified half-sine pulse, the approximate duration should be obtained first by controlling the carriage weight and the impact spring; then the approximate amplitude can be obtained by selecting the impact velocity of the carriage.

If appreciable rebound velocity of carriage does not occur, then Equations 9 and 10 are only approximate for determining amplitude, G, of the pulse. It is probably quicker to set up a trial shock, observe recorded G and t, and then make minor changes of W, k, and V_1. The necessary magnitude of the changes can be obtained from the following relationships:

$$G_2 = \frac{V_2}{V_1} \sqrt{\frac{k_2}{k_1}} \sqrt{\frac{W_1}{W_2}} G_1$$ (13)

$$G_2 = \sqrt{\frac{h_2}{h_1}} \left(\frac{k_2}{k_1}\right) \left(\frac{W_1}{W_2}\right) G_1 \quad \text{(free-fall only)}$$ (14)

$$t_2 = \sqrt{\left(\frac{W_2}{W_1}\right) \left(\frac{k_{e1}}{k_{e2}}\right)} \, t_1$$ (15)

where   Subscript 1 = First shock parameters

         Subscript 2 = Second shock parameters.

The relationships of Equations 13, 14, and 15 will be approximately true for any half-sine pulse generated, as long as there is some rebound and the pulse remains symmetrical.

Unfortunately, it is difficult to keep the same pulse shape when trying to change the amplitude or duration of a pulse. The cause of this difficulty is best shown by examining other shapes. The exact relationships of G, t, V_1, W, k, for other than half-sine pulses can be quite complicated.

Generating a Triangle Pulse

Figure 17 shows three typical load-deflection curves, any one of which can be used to generate a triangle pulse. For the 100-percent rebound condition, the spring constant increases with displacement while the spring constant for a half-sine pulse (same condition) is constant. This type of increase is characteristic of a rubber pad that is compressed far enough. The discontinuity in the jerk at the peak of the pulse never exists in actual testing. Therefore, the sharp peak at F_m in Figure 17 would be rounded off.

Generating a Haversine Pulse

The haversine pulse is a shock pulse that can be explained mathematically and also generated in the laboratory. This pulse shape is likely to occur most frequently on free-fall drop tables. The general mathematical expression for shock acceleration when the base acceleration level of this pulse is zero is

$$G = \frac{a}{g} = \frac{G_m}{2} \left(1 - \cos \frac{2\pi}{t_o} t\right) \qquad 0 \le t \le t_o.$$ (16)

The haversine pulse can occur because there is no discontinuity or rapid change in the jerk anywhere during the pulse. Figure 18 shows the requirements of the dynamic load-deflection curves for a haversine pulse caused by different impact materials.

In Figure 18 the curve for 50-percent rebound velocity is most typical of a material, such as rubber, that is bonded to the anvil. Unfortunately, this pulse shape cannot be predicted easily because of the flaring of the pulse shape at its start and end. The shape occurs when the right load-deflection curve appears for a given carriage weight, shock spring and impact velocity. When other load-deflection curves occur, the haversine pulse approximates either a half-sine pulse or a parabolic cusp pulse.

Generating a Parabolic Cusp

The parabolic cusp occurs when the dynamic spring constant of the shock spring is most nonlinear. The load-deflection curves combine characteristics of haversine pulse at the pulse start and end, and the triangle pulse near the shock peak. This pulse shape occurs most often when rubber pads are used which are not bonded to any surface and are highly compressed. The rapid change in spring constant occurs because the material is "bottoming out" during its deflection. Fastax movies taken during high-velocity impact indicate that the rubber flows outward as if it were a fluid. Figure 19 shows the appearance of three dynamic load-deflection curves for a parabolic cusp pulse. Butyl rubber and Ensolite pads are usually used to generate this pulse shape. The sharp peak at F_m will be smoothed out in the actual shocks generated.
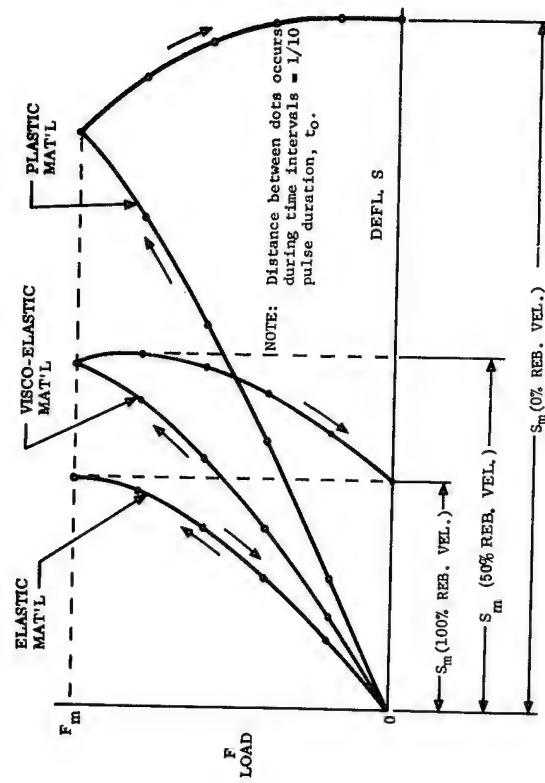
Figure 17. Dynamic Load-Deflection Curves of Various Materials
for a Shock Spring to Generate a Triangle Pulse

NOTE: Distance between dots occurs
during time intervals = 1/10
pulse duration, t_o.
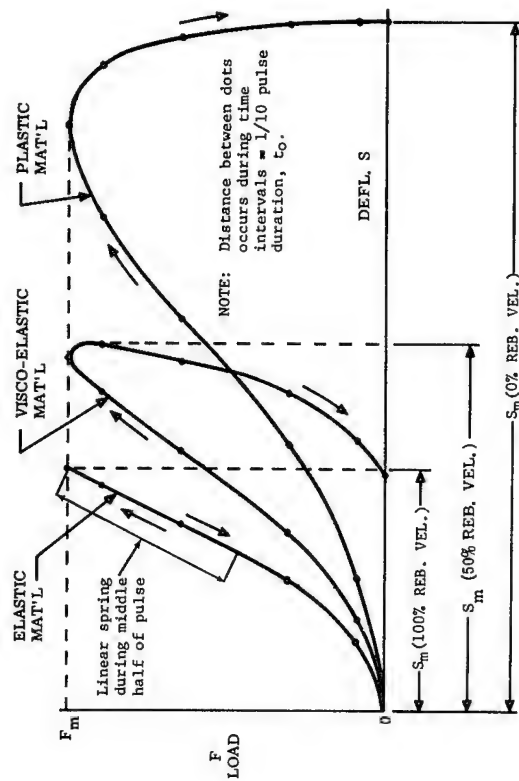
Figure 18. Dynamic Load-Deflection Curves of Various Materials for a Shock Spring to Generate a Haversine Pulse

PLASTIC MAT'L

VISCO-ELASTIC MAT'L

ELASTIC MAT'L

NOTE: Distance between dots occurs during time intervals = 1/10 pulse duration, t_o.

DEFL, S

S_m (0% REB. VEL.)

S_m (50% REB. VEL.)

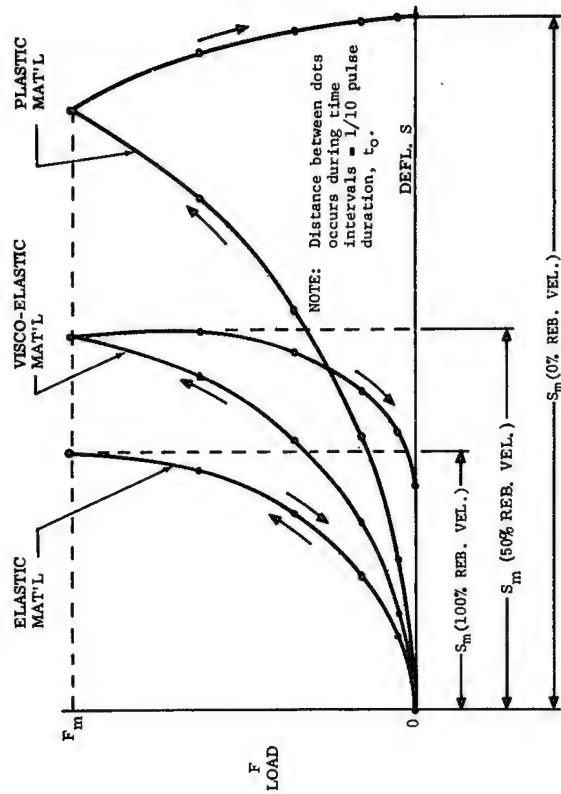S_m (100% REB. VEL.)

F_m

F LOAD

0

Figure 19. Dynamic Load-Deflection Curves of Various Materials for a Shock Spring to Generate a Parabolic Cusp Pulse

Generating Square, Saw-Tooth, Quarter-Sine Pulse Shapes

The remaining pulse shapes are generated most easily by crushing or shearing a material in such a way that only plastic deformation takes place. Practically no rebound will occur with these materials. The shock nomograph in the Appendix can be used to determine the necessary kinematic parameters to generate these pulses, or analysis can be done mathematically.

To generate a square pulse on impact machines, a material like lead or honeycomb can be crushed. Shearing a material like lead can also be used to generate the square pulse. The square pulse amplitude is controlled by the impact crush (shear) area, the crush strength of the material, and the carriage weight. The duration of the pulse is affected only by the impact velocity. The original thickness of a crushable material should be 1.2 times maximum shock displacement to allow room for the crushed material.

Saw-tooth pulses having a terminal peak can be generated by crushing a lead cone or a special honeycomb shape. In this case, the spring can be designed for one condition only; a certain impact velocity and a certain carriage weight. Changes in either of these conditions will produce an undesirable saw-tooth-type pulse. To generate a saw-tooth pulse any inelastic material having a dynamic load-deflection curve which follows a cubic equation will be suitable. When the base acceleration level of the pulse is negligible, the dynamic load deflection equation appears as

$$S = \frac{1}{2} g t_o^2 \left[\frac{F}{W} - \frac{1}{3G_m^2} \left(\frac{F}{W} \right)^3 \right] \qquad (17)$$

where

F = Variable crushing force during deformation (lbs)

G_m = Maximum faired amplitude

S = Shock displacement during crush (in)

g = Magnitude of acceleration due to gravity (in/sec/sec)

t_o = Shock pulse duration at zero line (sec)

W = Total carriage weight (lbs).

Initial peak saw-tooth pulses can be generated on an impact machine by shearing the cross section of a material that has constant shear strength. For a given pulse condition and carriage weight, the cross-sectional shape is dependent upon the impact velocity which must have some minimum but no maximum value. This particular pulse shape is not very much in demand.

To generate a quarter-sine pulse shape, honeycomb or lead can again be used. This pulse shape is much easier to generate than a saw-tooth pulse. Figure 20 shows the necessary shape of honeycomb to generate the quarter-sine pulse. The primary and residual shock spectra of a test item for this pulse is essentially everywhere equal to or greater than the primary and residual spectra from a terminal-peak saw-tooth pulse. While the honeycomb is crushing, it acts as an elastic spring which never recovers after the maximum displacement is reached. The dimension, b, angle, θ, and the crush strength, σ, of the prism determine the dynamic spring constant, k. Thickness, d, should be 1.2 times maximum shock displacement, S_m. All the formulas developed for a half-sine pulse will apply for this pulse until the peak acceleration occurs. The two important formulas are rewritten below:

$$G_m = V_i \sqrt{\frac{k}{gW}} \qquad (9)$$

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Equation 12 is rewritten as

$$t_o = \frac{\pi}{2} \sqrt{\frac{W}{kg}} \qquad (18)$$

since only half of the symmetrical pulse shape occurs. The value for k can be found from

$$k = 2\sigma b \tan \theta \qquad (19)$$

where σ, b, and θ are shown in Figure 20.

Impulse Machines

Considering shocks generated on impulse machines, the shock pulse is affected by the weight and stiffness of the moving parts, the force used to generate a changing velocity of the carriage, the maximum change in the velocity of the carriage, and the minimum stroke distance.

The weight of the moving parts determines the force necessary to produce a given amplitude. Force can be applied to the carriage by applying a difference in gas pressure across a piston. The net applied force on the piston can then be transmitted through a thrust column to an external carriage if desired. Otherwise, the piston itself becomes a carriage, as in an air-gun shock machine. A sled on a track works on the same principle. Force against the sled to cause its acceleration can be produced either by rockets or a water jet. For a given weight, the velocity change is a measure of the kinetic energy that must be generated from the initial potential energy. The shock displacement is the stroke length and is dependent on the velocity change of the pulse and the time of the pulse at the zero acceleration line.

For actuators, the pulse shape, magnitude, and duration collectively are determined by a particular pressure-volume relationship, and the adiabatic expansion rate of the gas. The rate of pressure drop with piston travel tends to produce a pulse approximating an initial peak saw-tooth or quarter-cosine pulse. The gas constitutes a so-called "shock spring" in this type of machine. To control the pulse shape, amplitude, and duration, it is therefore necessary to program the pressure acting against the piston face. Pressure programming occurs by metering the expanding gas through an orifice that changes area with time. Prediction of the correct programming for a gas is complex enough so that computer methods are employed to predict a certain pulse shape. Even then, the pulse shape is usually distorted because of the lack of usable data on the orifice efficiency and distortion of the internal members. Figure 21 shows an example of how a true adiabatic expansion of a gas must be modified by an orifice to generate an approximate haversine pulse in an air gun.

The pulse parameters are controlled mostly by rule-of-thumb techniques. If the orifice efficiency is assumed constant, a change in piston weight would require a change in both pressure and volume in the original breech chamber to keep a certain pulse shape. Generally, for a given weight, increase only in pressure tends to increase shock amplitude and change shape by shortening the fall time, while an increase only in volume tends to change shape primarily by increasing both rise and fall time. The symmetry of the pulse can best be changed by orifice programming.

At Sandia, the pulse shapes that can be best approximated on impulse machines are square, nonsymmetrical trapezoid, half-sine, haversine, and a nonsymmetrical shape that for the first half resembles the first half of a haversine, and for the last half resembles the second half of a half-sine pulse.

A special shock pulse, named a two-phase shock, consisting of a triangle spike followed by a half-sine drag phase pulse, can most easily be generated on an impulse machine. Since details of this pulse are covered in SCTM 61-62(73) they will not be discussed here.
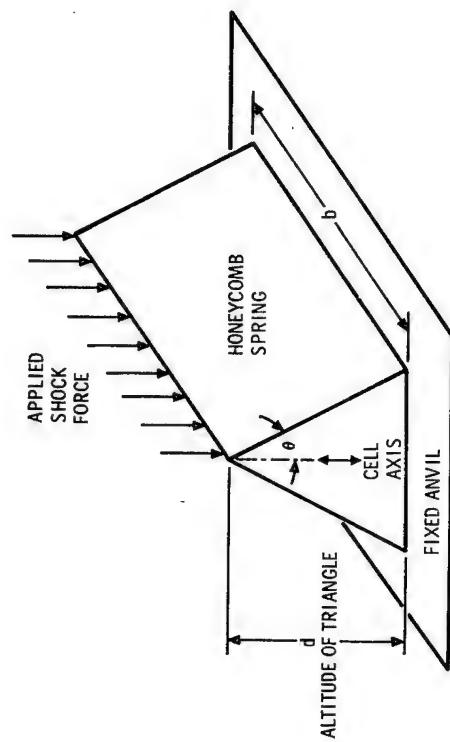
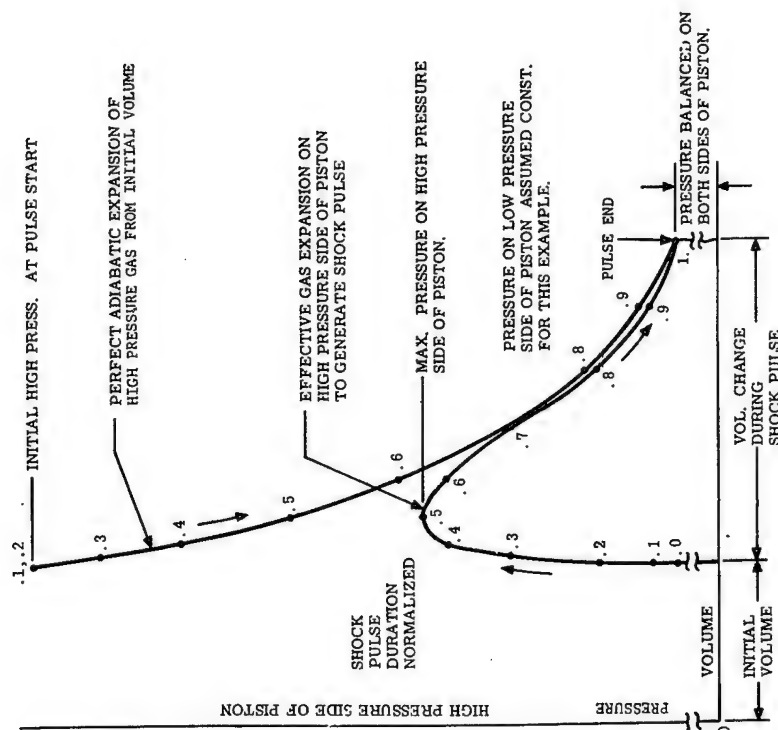Figure 20. Honeycomb Shape for Generating a Quarter-Sine Pulse

The effect of braking on the shock-pulse generation should be noted. On both impulse and impact shock facilities, some means of braking the carriage after the primary shock is needed. For impulse machines, brakes are needed to stop the carriage in a reasonable distance after the velocity has been generated. The braking force is usually applied even during the acceleration phase of the pulse, so it will affect the shock-pulse amplitude slightly for a given pressure setting. Impact machines should be equipped with brakes so as to catch the carriage upon rebound (whenever it occurs), to prevent the carriage from impacting a second time. A second impact occurring at the right interval of time after the first impact could cause more damage or malfunction of a test item than just a single impact. The effect of second impact on a test item is shown in Figure 22, where two different time intervals are considered. The effect of gravity is negligible for the number of cycles shown.

Fixture Design

The entire discussion to this point has only considered the input shock to the carriage containing the test item. The transmission of the shock from the carriage through the fixture to the test item should now be considered. The fixture incorporates all material between the carriage and the test item.

The primary function of the fixture is to adapt any test item to a particular carriage and to orient the test item so that the applied shock is in the right direction. The fixtures sometimes have to include thermal insulation whenever some mechanical shock tests are to be conducted at an extreme temperature. The fixture should be as rigid as practical so that it can be considered as part of the carriage. In general, recommended practices in the design of vibration fixtures also would apply to the design of a mechanical shock fixture. Quite frequently the same fixture can be used for both shock and vibration tests. In shock work, however, the fundamental axial resonant period of the fixture and also the loaded table should be less than one-half of the specified pulse rise time. This restriction is to insure that the transmitted shock through the fixture will come out in the same shape in which it entered. The fixture will respond to a shock just as any other spring-mass system would respond.

An accurate analytical solution of the fundamental natural periods of a fixture is not easily obtainable in a composite structure, which most fixtures are. By making some simplifications for each portion of the fixture, some good or acceptable approximations are possible. At least, the lower limit of possible natural periods can be determined.

Types of fixture assemblies in shock work will generally fall into a combination of one or more of three major categories of spring-mass systems. These categories are: plates, beams, and axial or column springs. Refer to SC-4452B(M), Appendix A, from which some of the information will be presented.

Rectangular Plates

The theory of vibration of plates is quite difficult and the results might be of limited value. The type of loading on the plate, the edge condition and the plate dimensions are the most important parameters affecting the natural period. Generally, the edge condition is assumed to be a combination of simply supported and pinned sides. The types of loading would vary from a negligible external load to a heavy external load distributed over a large portion of the plate.

The fundamental natural period of a round or square plate can be calculated from the following formula:

$$T = 1000\frac{a^2}{C} \sqrt{\frac{(1-\mu^2)w}{Eh^2g}} \tag{20}$$

Figure 21. Necessary Orifice Programming of Adiabatic Gas Expansion to Generate a Haversine Pulse in an Air Gun

where

T = Natural period, ms

a = Radius of round plate or edge of square plate, in.

μ = Poisson's ratio

w = Specific weight of plate material, lbs/in³

g = Acceleration due to gravity, in/sec²

E = Modulus of elasticity, lbs/in²

h = Plate thickness, in.

C = Constant for plate edge condition.

Values of C follow:

Condition	Square Plate	Round Plate
All edges fixed	1.654	0.469
Simple supported edge	0.910	0.237
Free edges	0.648	0.241
Free edge fixed at center	--	0.172

Beam

The fundamental natural period of a beam depends upon the end conditions and the type and location of any load. The general expression for a beam natural period can be approximated by

$$T = \frac{1}{C}\sqrt{\frac{w'L^4}{gEI}} \qquad (21)$$

where

T = Beam fundamental natural period (ms)

C = Constant depending on beam end conditions

$\left(\begin{array}{l}\text{for cantilever, } C = 560 \\ \text{for fixed-fixed end, } C = 3560 \\ \text{for simply supported, } C = 1570\end{array}\right)$

w' = Composite weight per unit length of beam (lbs)

L = Beam length (in)

g = Acceleration due to gravity (in/sec²)

E = Modulus of elasticity (psi)

I = Bending moment of inertia (in⁴).

From Equation 21 the natural period is dependent mostly upon the beam dimensions for a given loading, and upon the end conditions.
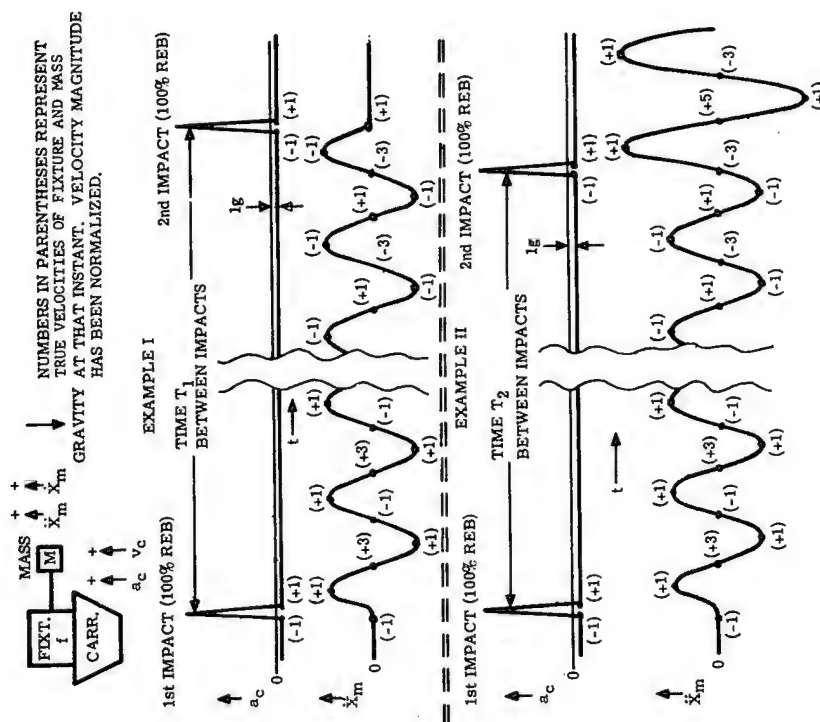

NUMBERS IN PARENTHESES REPRESENT TRUE VELOCITIES OF FIXTURE AND MASS AT THAT INSTANT. VELOCITY MAGNITUDE HAS BEEN NORMALIZED.

GRAVITY

MASS M
FIXT.
CARR.
$\ddot{X}_m$ $\dot{X}_m$
a_c v_c

EXAMPLE I

1st IMPACT (100% REB)   2nd IMPACT (100% REB)

TIME T₁ BETWEEN IMPACTS

EXAMPLE II

1st IMPACT (100% REB)   2nd IMPACT (100% REB)

TIME T₂ BETWEEN IMPACTS

Figure 22. Effect of 2nd Impact at Different Time Intervals Upon Acceleration Levels of Mass, m

Column

The natural period determination for this type of spring must take into account whether the spring's own weight can be neglected or not. For a uniform cross section the natural period of a spring can be generally written as

$$T = 2000 \pi \sqrt{\frac{W'L}{gAE}}$$ (22)

where all terms are as mentioned in Equation 21 except

W' = Sum of supported weight and effective weight of spring (lbs)

A = Cross-sectional area (in²).

The figure on page 3.A of SC-4452B(M) shows a quick means of determining the natural frequency or effective weight of a column fixture when the static weight of the spring must be added to the static weight of the load.

From the natural frequency, the natural period in milliseconds can be determined from

$$T = \frac{1000}{f}$$ (23)

At best, the above knowledge will only give a fixture designer a place to start his design. All the formulas assume that the carriage is very rigid and massive in comparison to the fixture. If the fixture is a significant percent of the carriage mass, then more complications enter into the picture.

Fixtures in shock work can be classified as adapter, box, upright, block, and standoff.

The adapter fixture is usually just a plate which can be used to fasten any test item to a particular shock machine.

The box fixture could be considered as several plates put together that are fastened on at least three sides. This type of fixture gives a good support to a small item in any one of six orientations. Considered as a column spring, it would probably have a much shorter natural period than if it were considered as a plate.

An upright fixture (sometimes referred to as a knee jig) would probably be considered as a beam for transverse natural period determination, a column for longitudinal natural period determination, and perhaps a plate if the vertical mounting surface was not heavily webbed on the other side from the mounting surface.

The block fixture is a column-type spring. It occurs most often when it contains potted-type components that are to be tested. The natural period can easily be determined from the figure on page 3.A in SC-4452B(M), if the respective weights of the test item and the block, and the depth of the block are known.

A standoff fixture is just a plate fastened to four column springs. (If the plate is extremely rectangular, then the plate would approximate a beam.) The natural period, assuming either type of spring, should be checked. This type of fixture is used to orient an odd-shaped test item in the most awkward direction for shock testing; i.e., the test item is suspended beneath the plate.

One interesting item in fixture design is apparent when weight on the fixture is negligible; the material is relatively unimportant. It can be shown that steel, aluminum, magnesium, and even wood, all have the same fundamental natural period for a given size and shape.

Bolts

Bolts used to fasten the fixture to the carriage must also be considered in fixture design. If bolts are assumed to take all the load, then the bolt can be considered as a weightless column spring. The nomograph shown on page 7.A of SC-4452B(M) can be utilized to determine natural frequencies of the bolt system. From the nomograph, one can see that the number of bolts and the bolt lengths (between head and engagement) are the most important parameters for changing the natural frequency. It is therefore recommended that the bolt length be kept as short as possible by counterboring the fixture. Any counterboring should allow for a reversal of fixture orientation to the carriage. It is also recommended that the shoulder of the fixture under the bolt head be at least two to four times the bolt diameter.

The actual load that occurs on a bolt during a shock test is difficult to determine exactly because the bolt and mating parts are both relatively elastic. A maximum load on the bolt will occur if the bolt is assumed to be rigid and the parts held together are elastic. Then the load is equal to the sum of the initial tightening tension and external loads. Rigid bolts and elastic plates do not occur often in fixture design. In present practice at Sandia, only the external load is considered, and the initial pre-tension load on the bolts is ignored when they are torqued to the value shown in the table on page 2.A of SC-4452B(M). All torque values shown in the table are sa.e for steel bolts threaded into the following material: steel, 1-1/2-bolt diameters; aluminum, 2-1/2- to 3-bolt diameters. Torquing of bolts provides for more standardization and uniformity in shock testing.

If straps are used to fasten a test item to the carriage, then the strap (like the bolt) must be considered as a spring.

For any fixture design, dynamic balance at the center of the carriage is desirable, to prevent the occurrence of undesirable torques to the carriage during the primary shock pulse in a shock test.

Shock-Test Instrumentation

Correct or good instrumentation in a shock test is most important. Transients occur in the electrical circuit from the mechanical shock transient. Therefore, it is the job of the instrumentation designers to assure test engineers that the electrical transients occurring during a mechanical shock truly represent the mechanical transient with a minimum amount of error. In SCTM 216-62(73) and SCTM 217-62(73), details of two types of shock-test instrumentation circuits used at Sandia are discussed, involving the pickup selection, its associated circuitry, calibration, and test restrictions. It is stressed here though, that instrumentation used to monitor shock tests should be adequately and correctly calibrated for the transient work. Errors in instrumentation in shock work can best be described in some such manner as "accurate to ±10 percent in 95 percent of the shocks monitored."

Shock-Pulse Interpretation

Interpretation of a recorded shock pulse is a phase of shock testing where large differences of opinion can exist between individuals or companies. Three people not too familiar with shock testing could look at a particular shock-pulse picture and arrive at three different conclusions (all wrong) about the generated shock-pulse conditions and the effect of the shock upon the test item.

To create some standardization in nonsmooth generated shock-pulse interpretation, Sandia Corporation's standard method for interpreting shock pulses is shown in SC-4452B(M). A diagram of this method is reproduced here in Figure 23. Quite often the shock nomograph in the Appendix can be used to catalog generated pulse shapes in terms of finding deviations from a test request. When the generated pulse shape is a deviation, and has been cataloged, then the
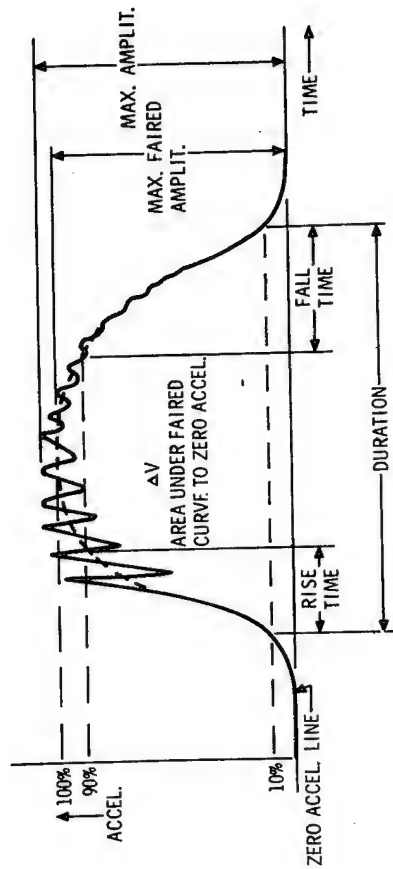
Figure 23. Standard Pulse Interpretation

shock spectrum of that cataloged generated pulse shape can be examined and compared to the shock spectrum for the requested pulse, to determine whether the deviation is acceptable or not. Cataloging a generated shock pulse shape can best be done by looking at its area, duration, rise time, and jerk, and comparing these parameters to a standard pulse. All information except jerk can be obtained from the nomograph. Jerks of recorded pulses can be compared to standard pulses by noting the following conditions for each pulse shape:

Half-sine -- Jerk is a maximum close to the zero line, 10 to 20 percent of maximum faired amplitude.

Triangle -- Jerk is constant approximately between the 10- to 90-percent faired amplitude level.

Haversine -- The jerk is a maximum approximately at the 50-percent faired amplitude level.

Parabolic -- The jerk is a maximum approximately 80- to 90 percent of the cusp faired amplitude level.

Fairing of the pulse amplitude is a graphical smoothing of the recorded pulse to eliminate hash, ringing, or undesirable higher frequencies which are unimportant in a particular test. In general, the fairing line is drawn so that equal plus or minus amplitudes of the higher frequencies will occur on either side of the line. A change in the faired shape to determine the duration is never attempted, however, unless the pulse shape is very distorted or complex in appearance. The maximum faired amplitude and the maximum amplitude of a recorded pulse are not the same when a higher frequency signal is superimposed on the basic pulse. The meanings of the different nomenclature are shown in Figure 23. Sometimes, comparing the same shock pulse when filtered through various low-pass filters, each having different cutoff frequencies, gives a good indication of how to fair some of the pulses generated which have a lot of "trash." Whenever a filter is used, however, the recorded pulse will experience a lagging phase shift with respect to the unfiltered recorded pulse. This phase shift must be considered if the response of a test item is being monitored simultaneously with the input shock.

The zero acceleration line of a pulse cannot be determined from a picture of just one trace-sweep. A recorded shock pulse needs to be evaluated in relation to how the actual basic mechanical pulse should look for a given facility to determine zero acceleration line. In all vertical motion facilities, base acceleration line and zero acceleration line of a shock pulse will differ by 1 g or more. If the difference in the base acceleration line and the zero acceleration levels in certain tests is ignored, trouble can occur. If the carriage is being highly accelerated in the opposite direction at the time the primary shock pulse is applied, a serious undertest to the test item can result. Error will be introduced into measured structural load levels inside the test item during the primary shock if the change in the acceleration from the base acceleration level is used as shock acceleration, and the shift in the base acceleration level from the zero acceleration level is not considered. The actual amplitude of shock-motion response is dependent upon the base acceleration level before and after the primary pulse is applied.

Figures 24 to 28 show the entire acceleration-time histories of different machines at Sandia from the time the carriage is released until it comes to rest.

Whenever possible, pulse time progression should travel from left to right on a recorded shock picture.

Sometimes, when shock tests are conducted on a free-fall machine, the recorded shock picture will indicate a sizable acceleration level in the negative, or downward, direction after the pulse is completed. This negative undershoot is the result of the RC time constant of the monitoring circuit being too short in relation to the pulse duration. This effect occurs on pulses recorded using piezoelectric-type accelerometers. The effect of this undershoot is to reduce the amplitude and shorten the duration of the recorded pulse with respect to the pulse actually produced by the shock machine. For a recorded half-sine pulse that
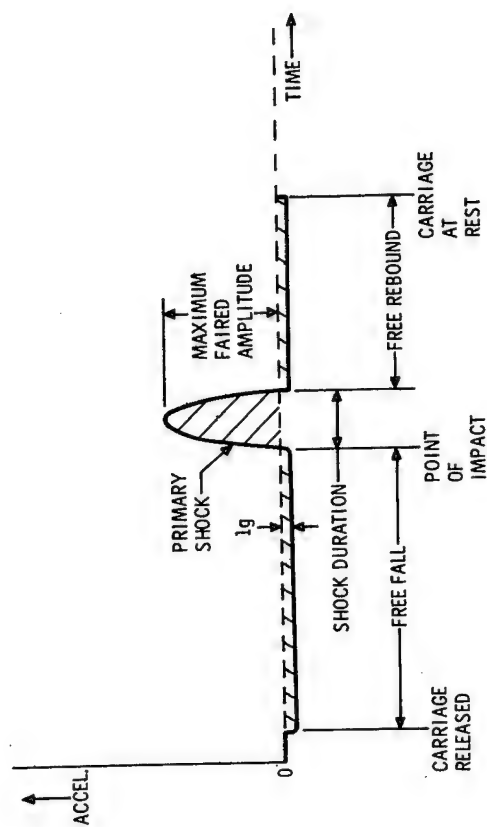
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Figure 24. Impact Shock - Free Fall Machine (with Rebound)

ACCEL.

PRIMARY SHOCK

MAX. FAIRED AMPLIT.

1g

0

CARRIAGE RELEASED

DURATION

POINT OF IMPACT
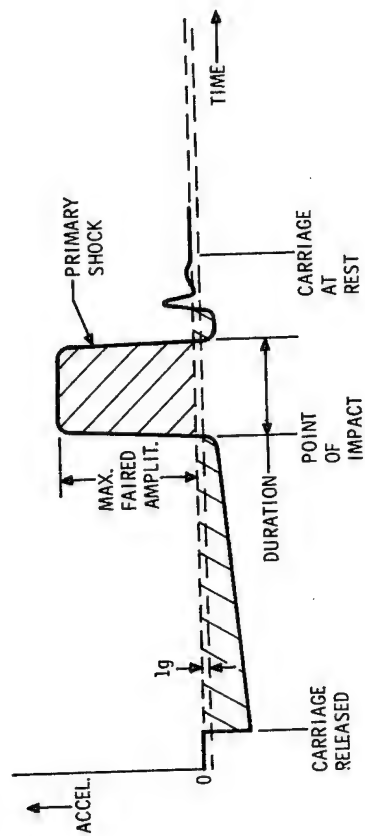
CARRIAGE AT REST

TIME

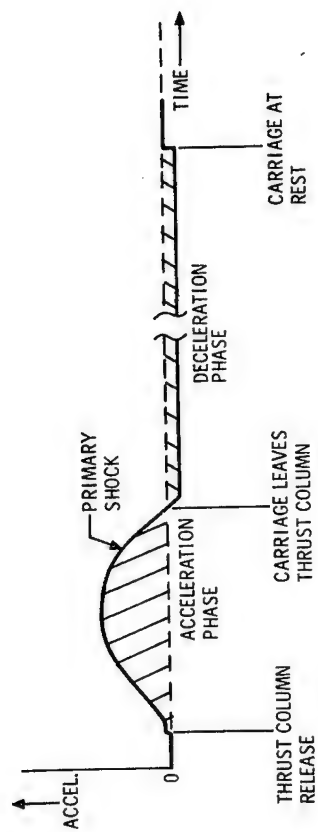Figure 25. Impact Shock - Accelerated-Fall Machine (Low Rebound)

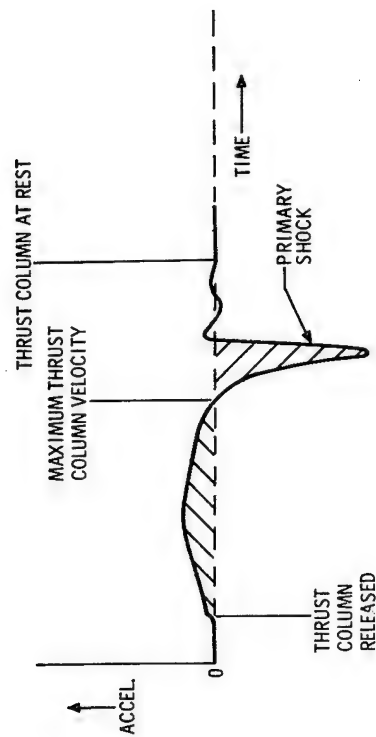Figure 26. Impulse Shock – Actuator (Free Carriage)

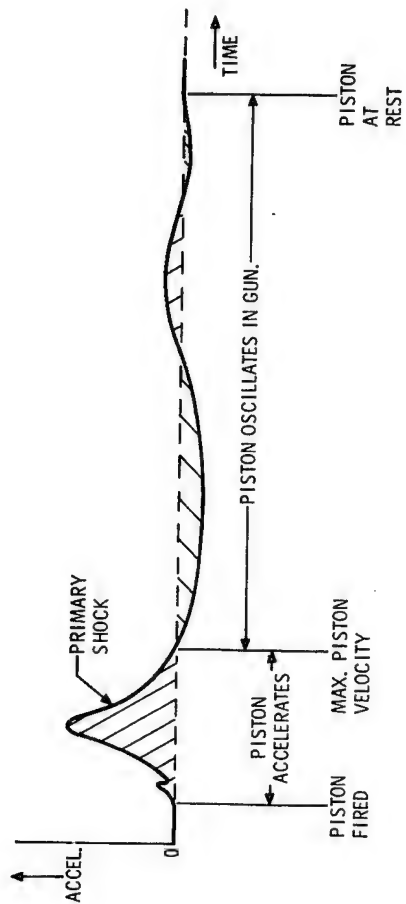Figure 27. Impact Shock - Actuator (Thrust Column Carriage)

Figure 28. Impulse Shock - Air Gun (Pinton)

has an undershoot of 10 percent of the recorded pulse peak faired amplitude, the recorded pulse peak amplitude would be 6 percent lower than the generated pulse peak amplitude; and the duration of the recorded half-sine pulse would be off 4 percent from the generated duration. This magnitude error occurs when the ratio of RC time constant to pulse duration is approximately 20. This undershoot effect is different from the zero shift effect which can also occur. The zero shift effect is a phenomenon which is caused by the accelerometer itself receiving shock loads which sometimes cause the apparent zero acceleration line to radically change. When zero shift occurs, neither the amplitude nor the duration of the shock pulse can be accurately determined.

When pulses monitored by a piezoelectric accelerometer are filtered, the filter cutoff frequency should not be too low in regard to the pulse duration. Figure 29 shows the distortion effect of too much filtering on a recorded shock pulse when the ratio of RC time constant of monitoring circuit to pulse duration is approximately 5.

When unbonded strain-gage accelerometers are used to monitor the pulse, the natural frequency and damping ratio of this type of accelerometer should be known. Monitoring the same shock, the use of strain-gage accelerometers with different natural frequencies will cause the recorded shock pulse to appear different. Figure 30 shows a typical shock pulse monitored by three different unbonded strain-gage accelerometers having different natural frequencies. The maximum recorded amplitude in each case is considerably different while the maximum faired amplitude is the same. The differences in recorded pulses occur from different pickups because the frequency of the ringing is either greater or less than the particular accelerometer's natural frequency. If the ringing frequency is higher than accelerometer natural frequency, its amplitude is attenuated. For some accelerometers, if the ringing frequency is equal to the natural frequency, amplification of the ringing signal might occur. The effective damping of the pickup will also affect amplitude of ringing frequency.

When evaluating shock pictures, it is desirable to observe the complete primary shock pulse. This requires external triggering when the readout device is an oscilloscope. A tape recording or an oscillograph-type recording does not require any triggering. In fact, tape recording of shock pulses is becoming a good way to monitor shock tests, expecially those which can be conducted only once. On tape, the unfiltered shock pulse can be recorded and then played back at different speeds, either filtered or unfiltered. For shock tests where several channels of information are desired, any channel can be played back and examined without interference from any other channel.

It is very important that the entire instrumentation circuitry from the accelerometer to the readout device be correctly calibrated. One way to check the accuracy of a recorded shock picture is to monitor the shock by using two pickups, preferably or different types, mounted at different locations on the test fixture. The reason for checking picture accuracy is that in accelerometer calibration there is no standard with which to check the accelerometer results except, possibly, by mechanical impedance techniques. In the shock tests, if the recorded pictures (using two accelerometers) show considerable differences in shock amplitude, which is the most difficult parameter to check accurately, a check of the pulse area will indicate which picture is in error. If the two recorded amplitudes are slightly different, interchanging the two pickups and repeating the shock test will reveal whether the amplitude error is due to accelerometer calibration or to accelerometer location due to an unfortunate fixture design. If a strain-gage accelerometer and piezoelectric accelerometer were found suitable to monitor the same test, and were both accurately calibrated, the amplitude of one trace in the recorded pulse might be different from the amplitude of the other. In addition, the duration would be different if the amplitudes were different, but the areas under both curves would be the same. This effect might occur because one accelerometer is mechanically undamped (filtered) while the other accelerometer is essentially undamped in addition to other monitoring circuit differences.

Another, although more difficult, way to check pulse accuracy is to electronically double-integrate the recorded acceleration-time pulse and compare it to a monitored displacement-time pulse. In most shock machines, this latter method is not very feasible, however.

For all pictures:

Vertical calibration $\quad$ g/cm = 10
Horizontal calibration $\quad$ msec/cm = 5

(a)

Unbonded Strain Gage Accelerometer Having Natural Frequency = 565 cps

(b)

Unbonded Strain Gage Accelerometer Having Natural Frequency = 1850 cps

(c)

Unbonded Strain Gage Accelerometer Having Natural Frequency > 2000 cps

Figure 30. Pictures of Shock Pulse Utilizing Unbonded Strain Gage Accelerometers Having Different Natural Frequencies

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$$\frac{\text{Time Constant}}{\text{Pulse Duration}} - \frac{\pi RC}{t_o} = 5$$

For all pictures:

Vertical calibration $\quad$ g/cm = 8.5
Horizontal calibration msec/cm = 5

(a)

Low pass filter $\quad$ = 5000 cps
Corrected amplitude = 31 g
Corrected duration = 14.3 msec

(b)

Low pass filter $\quad$ = 500 cps
Corrected amplitude = 32 g
Corrected duration = 14.3 msec

(c)

Low pass filter $\quad$ = 100 cps
Corrected amplitude = 31 g
Corrected duration = 13.7 msec

(d)

Low pass filter $\quad$ = 50 cps
Corrected amplitude = 26 g
Corrected duration = 18.7 msec

Figure 29. Effects of AC Coupling and Filters in Piezoelectric Accelerometer Circuitry for a Given Shock Pulse

APPENDIX A

Shock Spectra

Shock spectra (Figures A-1 to A-8) are shown for undamped, single-degree, linear spring-mass systems that are subjected to the standardized pulse shapes shown in Figure 6 of the text. The duration of each pulse is measured at the zero reference line. (Corrections for pulse durations measured at the 10-percent amplitude level can be made by using the nomograph shown in Figure A-9.)

The primary shock spectrum for a given input pulse is a plot of maximum positive* accelerations experienced by each mass of a number of spring-mass systems (having different natural frequencies) versus the natural frequencies of these systems. It can also be a plot of the maximum positive accelerations experienced by the mass of one spring-mass system versus single shock pulses of certain shape and amplitude having various durations.

The residual response spectrum is a plot of the maximum peak accelerations occurring after a pulse is over, for several spring-mass systems subjected to one pulse, and for one spring-mass system subjected to any one of several pulses.

Mechanical Shock Nomograph

In general, a nomograph offers three major advantages within allowable limits of accuracy:

1. The saving of time in calculating equations that are highly repetitive

2. The elimination of mistakes that are likely to occur in longhand computations

3. The reduction in overhead expense for routine tests or control work by using lower salaried technicians in place of highly paid personnel.

In shock testing, the nomograph shown in Figure A-9 is useful in permitting rapid calculations and checks of shock-pulse requirements to see if they can be conducted on available equipment. The nomograph permits cross checking of test setups and instrumentation calibration to see that the test is being conducted in accordance with the consultant's wishes.

The nomograph is also quite useful in converting the duration measured at the base of the pulse to that measured at the 10-percent faired amplitude level. Another use is in determining the rise time (of the standardized pulse shapes) between the 10- to 90-percent faired amplitude levels. The velocity change, ΔV, of all the standard pulses can be calculated when the maximum faired amplitude, G, and duration, t, are known.

The nomograph can be used to catalog recorded shock pulses to the nearest standard pulse shape. After cataloging, the approximate shock spectra for the recorded pulse can be determined from the appropriate figure selected from Figures A-1 to A-8.

*Positive direction - same direction as input shock direction.

The area under the shock pulse can be easily predicted and checked. The area or subarea of any curve can be easily obtained by integrating the equation of the curve, if known. For pulses that cannot be simply expressed mathematically, graphical integration can be tried. Graphical integration is done by approximating enough of the subareas of the pulse by trapezoids and triangles, the areas of which are easily determined. Areas can also be determined by the use of mechanical integrators, such as planimeters. Electronic integrators can also be easily used if the pulse is recorded on tape.

The importance of good and accurate instrumentation cannot be stressed too greatly, for, by the astute juggling of instrumentation circuit characteristics, the recorded pulse can be so distorted from the actual pulse generated that no similarity exists between the two. Thus, the inability of the machine mechanically to produce a requested shock exactly, the inability of instrumentation to record a pulse accurately, and the inabilities of different individuals to interpret the pulse exactly, can cause difficulties between different companies in trying to generate a requested shock pulse. Problems will occur, especially when the test item happens to be marginal and passes a shock test on one machine and fails under the supposedly same shock test at another shock machine either in the same company or another company. Messrs. W. J. Sieger and E. H. Copeland showed in their paper, "Uniformity in Shock Testing," that three different shock machines at Sandia could produce a 180-g, 11-millisecond shock pulse approximating a half-sine shape. These particular shock pulses were analyzed on an analog computer. The recorded pulse data was the input to the analog computer. By varying the time constants of the analog circuits (which simulated test items having different natural periods), the shock spectra from the input pulses were obtained. A comparison of shock spectra was also made of the response of the same analog circuits to a half-sine input of the same durations and amplitudes as the recorded pulses. Results indicated that the recorded pulse-shape deviations would cause some test items to be undertested, in regard to a half-sine pulse, and other test items to be overtested, depending upon the test item's natural frequency. A lot of research is still needed in the field of shock spectra.

Conclusions

In shock testing, as now practiced, there are four major areas which need continual improvement. These are:

1. Obtaining an adequate and satisfactory specification for shock tests

2. Accurately generating a requested shock pulse, in order to produce a desired response in a test item to assure its satisfactory performance in the field

3. Possessing adequate calibrated instrumentation, to insure that the recorded shock pulse most nearly represents the actual generated shock pulse

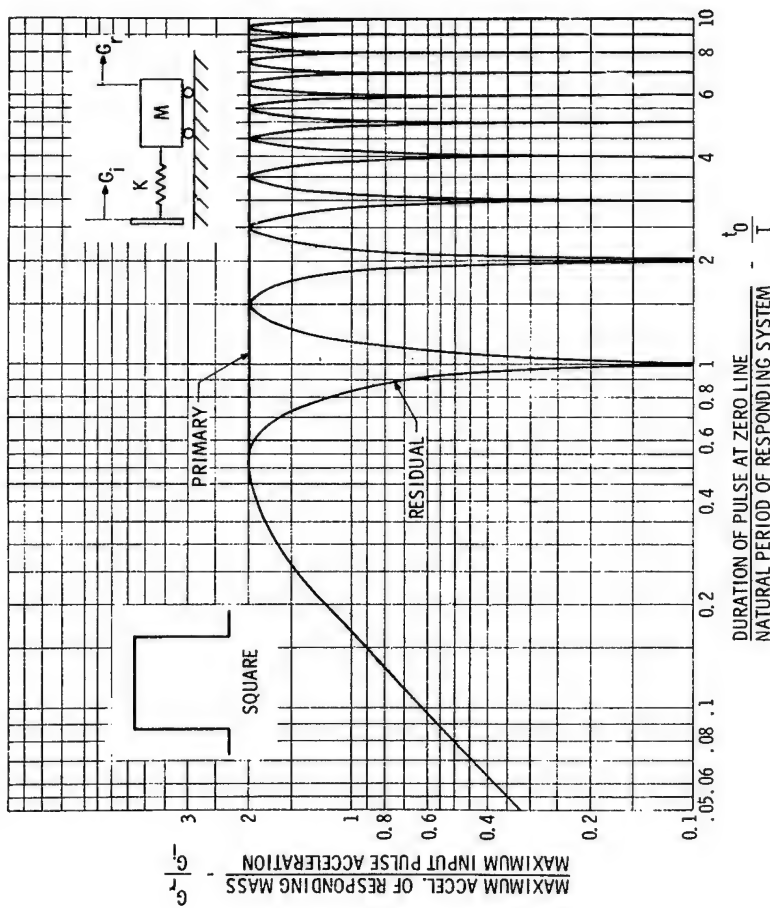4. Correctly interpreting both the input shock and the response of the test item to the shock.

Figure A-1. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Square Pulse
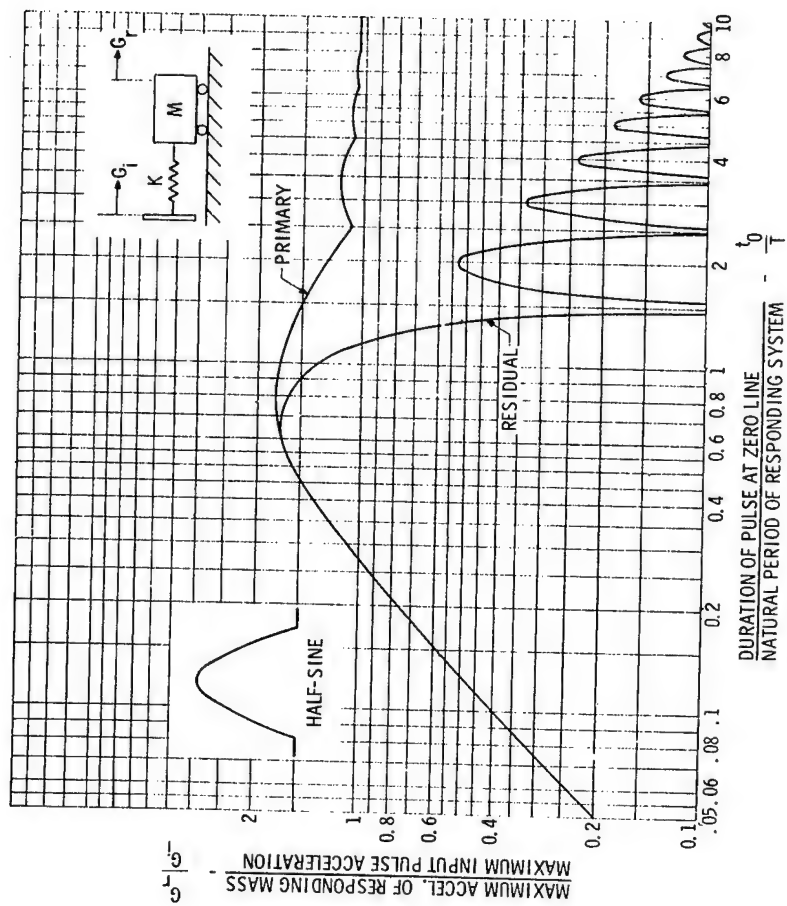
Figure A-2. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Half-Sine Pulse
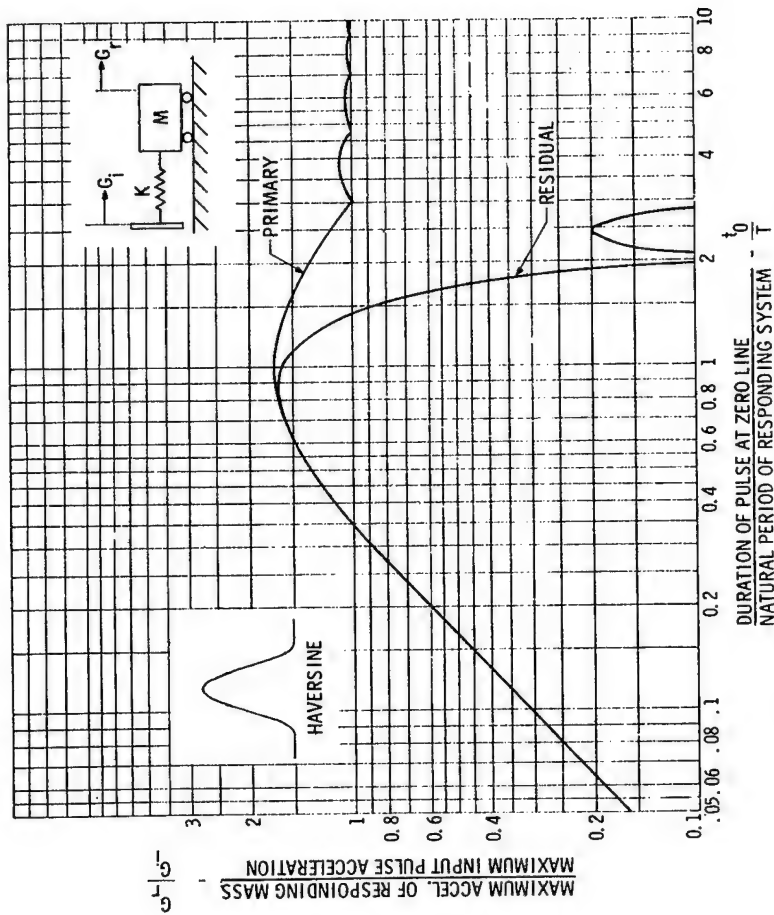
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Figure A-3. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Haversine Pulse
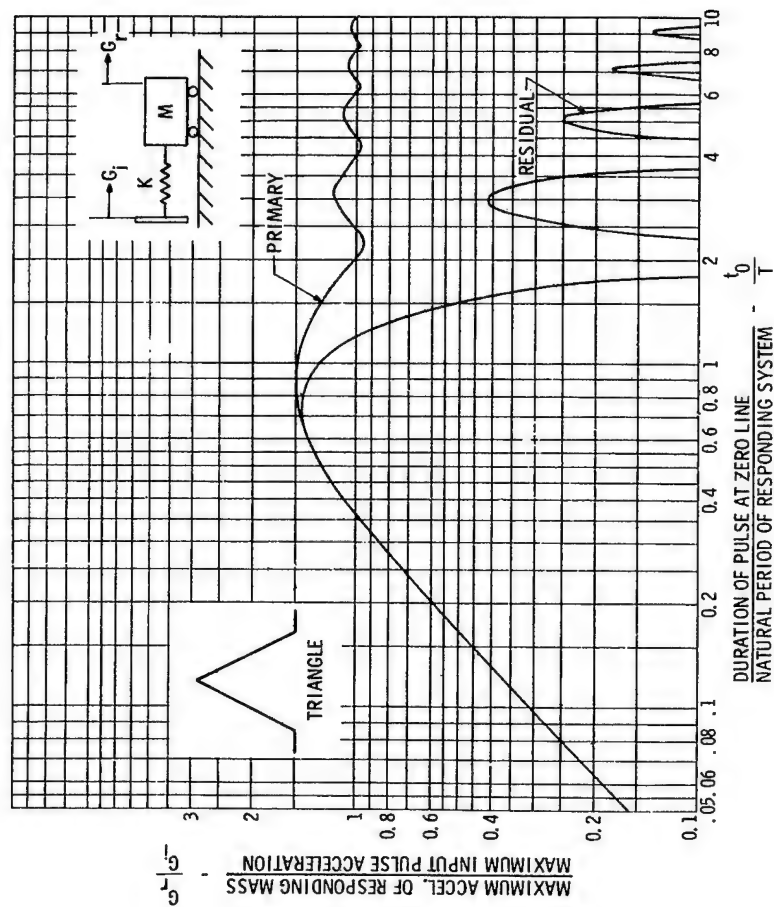
Figure A-4. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Triangle Pulse
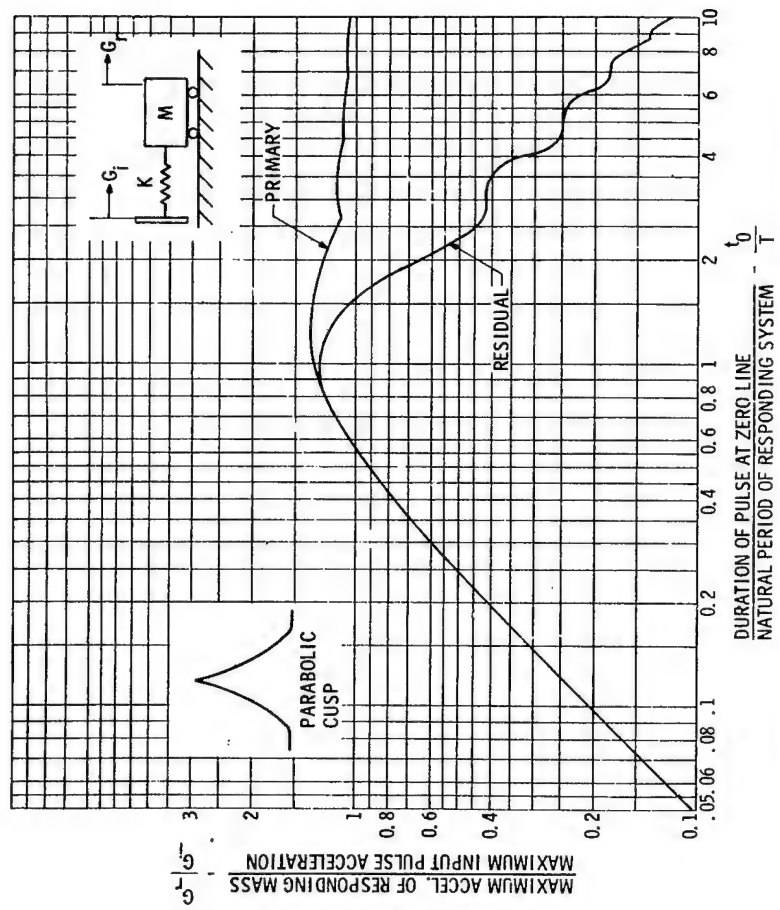
61

Figure A-5. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Parabolic Cusp Pulse
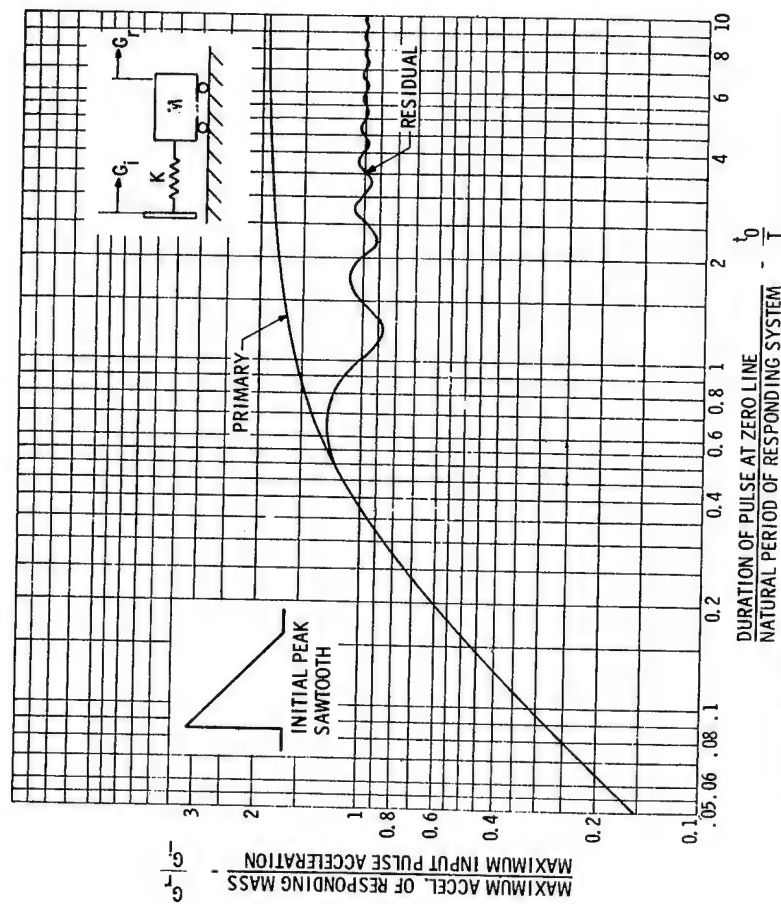
Figure A-6. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Initial Peak Sawtooth Pulse
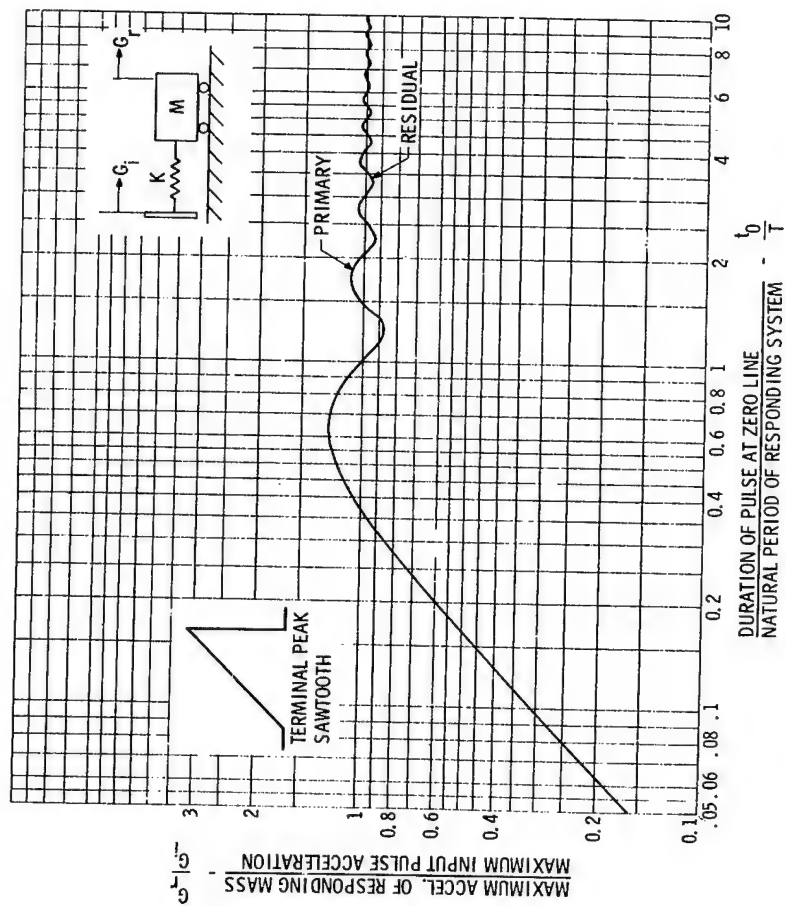
Figure A-7. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Terminal Peak Sawtooth Pulse
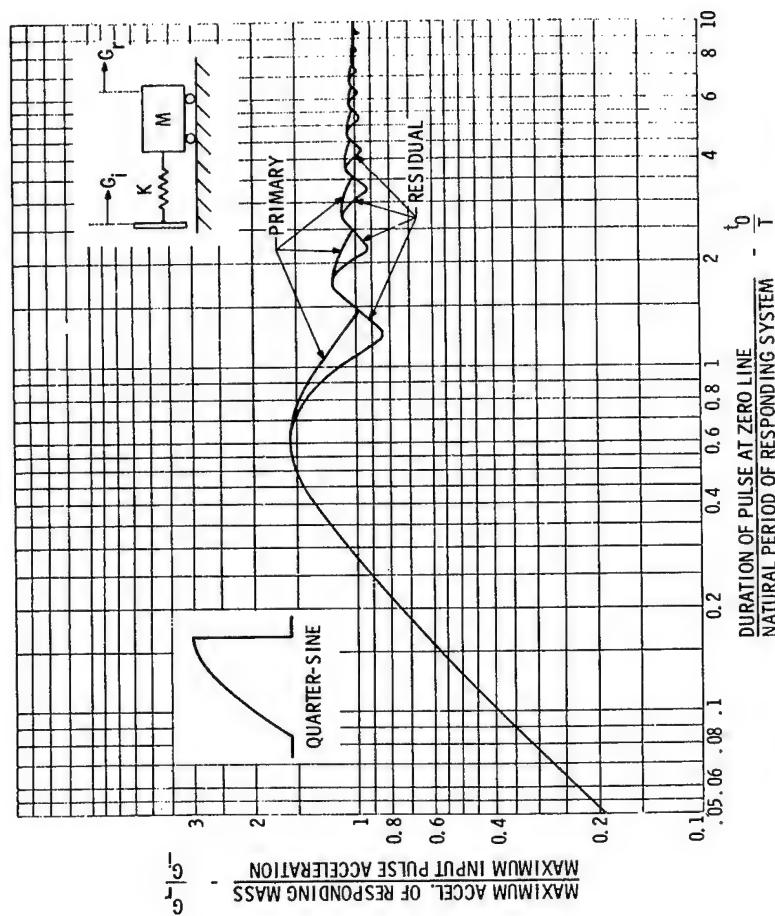
Figure A-8. Primary and Residual Shock Spectra for Undamped Single Degree Freedom Linear System Subjected to Quarter-Sine Pulse

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Figure A-9. Sandia Corporation Mechanical Shock Nomograph

Kinematic Relationships (Mathematical Analysis)

Example No. 1 Half-Sine Pulse, 100 g, 20 ms (zero line)

If the base acceleration level at the beginning of the primary pulse is zero, the general acceleration-time equation can be written as

$$a = gG_m \sin\left(\frac{\pi}{t_o} t\right) \qquad\qquad o \leq t \leq t_o \qquad\qquad (A-1)$$

where

a = Acceleration at any time, t (ft/sec^2)

g = Magnitude of acceleration due to gravity (ft/sec^2)

G_m = Ratio of maximum-faired acceleration to acceleration due to gravity (numerically it is equal to maximum faired acceleration measured in "g" units)

t = Time (sec)

t_o = Pulse duration, zero reference line (sec).

Integrating Equation A-1, the general equation of velocity is

$$v = -\frac{gG_m t_o}{\pi}\left[\cos\left(\frac{\pi}{t_o} t\right)\right] + C_1 \qquad\qquad (A-2)$$

where

v = Velocity at any time, t (ft/sec)

C_1 = Constant of integration.

At this point C_1 can be determined only when a type of shock machine is arbitrarily selected.

Case I. Impulse Machine -- Using an impulse machine, the initial conditions are known from which C_1 can be determined; namely,

at $t = 0$, $v = 0$.

Substituting the values into Equation A-2,

$$C_1 = \frac{gG_m t_o}{\pi}$$

or Equation A-2 becomes

$$v = \frac{gG_m t_o}{\pi}\left[1 - \cos\left(\frac{\pi}{t_o} t\right)\right] \cdot \qquad\qquad (A-3)$$

and at $t = t_o$, the end of the pulse

$$v = V_e$$

$$= \Delta V$$

$$= \frac{2gG_m t_o}{\pi} \qquad (A-4)$$

where

V_e = Velocity at stroke end (fps).

ΔV = Velocity change; total area under pulse (fps).

Integrating Equation A-3 and knowing that at $t = 0$, $s = 0$, the general equation for displacement is

$$s = \frac{gG_m t_o}{\pi} \left[t - \frac{t_o}{\pi} \sin\left(\frac{\pi}{t_o} t\right) \right], \qquad (A-5)$$

and at $t = t_o$,

$$s = S_m = \frac{gG_m t_o^2}{\pi} \qquad (A-6)$$

where

s = Shock displacement at any time, t (ft)

S_m = Maximum shock displacement (ft).

From Equations A-1, A-3, and A-5, as time increases during the shock, three curves (acceleration, velocity, and displacement) of carriage motion are generated simultaneously.

For the example of a 100 g, 20 ms pulse, Equations A-1, A-3, and A-5 can be written as

$$a = 3220 \sin (157 t)$$

$$v = 20.5 [1 - \cos (157 t)]$$

$$s = 20.5 [t - 0.00636 \sin (157 t)] .$$

The curves of these three equations are shown in Figure 11a.

Case II. Impact Machine -- If Example No. 1 is conducted on an impact machine, different initial conditions exist for determining C_1 of Equation A-2. At $t = 0$

$$v = V_i$$

which, by substituting into Equation A-2, indicates

$$C_1 = V_i + \frac{gG_m t_o}{\pi}$$

or, in general

$$v = v_i + \frac{gG_m t_o}{\pi} \left[1 - \cos\left(\frac{\pi}{t_o} t\right) \right] \qquad (A-7)$$

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where

V_i = Impact velocity (fps).

Integrating Equation A-7 now and assuming $s = 0$ at $t = 0$,

$$s = V_i t + \frac{gG_m t_o}{\pi} \left[t - \frac{t_o}{\pi} \sin\left(\frac{\pi}{t_o} t\right) \right] . \qquad (A-8)$$

From Equations A-7 and A-8, v and s at any time, t, can only be determined when the amount of rebound is known. The amount of rebound determines the necessary V_i in Equation A-9 to produce a desired pulse area, ΔV.

$$\Delta V = |V_i| + |v_r| \qquad (A-9)$$

where

v_r = Velocity of rebound (fps).

For the condition of Example No. 1, three possibilities will now be assumed.

1. $V_r = 0$ (Occurs for 0-percent rebound)

2. $|v_r| = |V_i|$ (Occurs for 100-percent rebound)

3. $|v_r| = \frac{1}{2} |V_i|$ (Occurs for 50-percent rebound velocity).

From 1, $V_r = 0$ at $t = t_o$. Therefore, from Equation A-7

$$V_i = - \frac{2gG_m t_o}{\pi} .$$

Then, from Equation A-9, for no rebound

$$|V_i| = \Delta V = \frac{2gG_m t_o}{\pi} . \qquad (A-10)$$

Substituting the numbers for the given example into Equation A-10

$$V_i = - \frac{2}{\pi} (32.2)(100)(0.020)$$

$$V_i = -41 \text{ fps.}$$

Equations A-7 and A-8 become for the example

$$v = -20.5 [1 + \cos (157 t)]$$

$$s = -20.5 [t + 0.00636 \sin (157 t)]$$

while

$$a = 3220 \sin (157 t) .$$

The curves of these three equations are shown in Figure 11b.

For 2, when 100 percent rebound occurs

$$|v_r| = |V_i| .$$

$$\Delta V = 2 |V_i| \quad \text{from Equation A-9.}$$

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Therefore, from Equation A-7

$$v_i = -\frac{g G_m t_o}{\pi}$$

= -20.5 fps for the given example.

Equations A-3, A-7, and A-8 now become for the example

v = -20.5 cos (157 t)

s = -0.131 sin (157 t)

a = 3220 sin (157 t).

The curves of these three equations are shown in Figure 11c.

For condition 3, where 50 percent rebound velocity of the carriage occurs, the equations of velocity and displacement will be developed.

It should be obvious now that ΔV for any half-sine pulse can be written

$$\Delta V = \frac{2g}{\pi} G_m t_o .$$

Now, for the third impact condition

$$\Delta V = \frac{3}{2} |v_i|$$

Therefore

$$|v_i| = \frac{2}{3} \Delta V,$$

or observing the sign convention

$$v_i = -\frac{4}{3} \frac{g}{\pi} G_m t_o .$$

Therefore, from Equation A-7

$$v = -\frac{4g}{3\pi} G_m t_o + \frac{g}{\pi} G_m t_o \left[1 - \cos\left(\frac{\pi}{t_o} t\right)\right]$$

(A-11)

$$= \frac{g}{\pi} G_m t_o \left[-\frac{1}{3} - \cos\left(\frac{\pi}{t_o} t\right)\right] .$$

Integrating Equation A-11 and letting s = o at t = o

$$s = \frac{g}{\pi} G_m t_o \left[-\frac{t}{3} - \frac{t_o}{\pi} \sin\left(\frac{\pi}{t_o} t\right)\right] .$$

(A-12)

Substituting 100 g and 20 ms into equations A-1, A-11, and A-12 the corresponding kinematic equations of motion become

a = 3220 sin (157 t)

v = -6.84 - 20.5 cos(157 t)

s = 20.5 [-0.333 t - 0.00636 sin (157 t)] .

A plot of the above three equations for 50 percent rebound velocity is shown in Figure 12, with a duplication of Figures 11b, and 11c.

Example No. 2 -- Square Pulse, 100 g, 20 ms

The same techniques would be used in this example as in Example No. 1, so only a summary of pertinent equations will be listed below.

For an impulse machine where the initial velocity is zero,

$$a = g G_m$$

$$v = g G_m t \qquad 0 \leq t \leq t_o$$

$$\Delta V = g G_m t_o$$

$$s = \frac{1}{2} g G_m t^2$$

$$S_m = \frac{1}{2} g G_m t_o^2$$

(A-13)

For Example No. 2

a = 3220

v = 3220 t

s = 1610 t².

These three curves are plotted in Figure 13a.

For an impact machine, the general equations are

$$a = g G_m$$

$$v = V_i + g G_m t$$

$$s = V_i t + \frac{1}{2} g G_m t^2.$$

(A-14)

For a no-rebound machine the magnitudes of ΔV and S_m will be the same as for impulse machines. For a perfect-rebound machine, the general equations for the example become

a = 3220

v = -32.2 + 3220 t

s = -32.2 t + 1610 t².

The three curves are plotted in Figure 13c.

Example No. 3 -- Haversine Pulse, Low Amplitude, Long Duration

The magnitudes of acceleration for this pulse are assumed to be so low that on a free-fall machine the effects of gravity during the pulse must be considered.

The general acceleration equation for a haversine pulse having initial acceleration can be written as

$$a = \left(\frac{g G_m - 2a_o}{2}\right) \left[1 - \cos\left(\frac{2\pi}{t_o} t\right) + a_o\right] \qquad 0 \leq t \leq t_o$$

(A-15)

where

a_o = Initial carriage acceleration at impact (ft/sec^2).

Equation A-15 is integrated and the impact velocity (the instant the carriage hits the shock spring) is known to obtain

$$v = \left(\frac{gG_m - 2a_o}{2}\right)\left[t - \frac{t_o}{2}\sin\left(\frac{2\pi}{t_o}t\right)\right] + a_o t + V_i \ . \tag{A-16}$$

Equation A-16 is integrated and the shock displacement is measured from the instant the carriage touches the shock spring, to obtain

$$s = \left(\frac{gG_m - 2a_o}{2}\right)\left\{\frac{t^2}{2} + \left[\frac{t_o}{2\pi}\right]^2 \left[\cos\left(\frac{2\pi}{t_o}t\right) - 1\right]\right\} + \frac{1}{2}a_o t^2 + V_i t \ . \tag{A-17}$$

Figure 14 shows how Equations A-15, A-16, and A-17 appear when good rebound occurs.

Proof of Equation No. 6 of Text

Referring to Figure 15, the loss in potential energy of the system can be equated to the gain in kinetic energy of the system at the moment of impact. Since the suspension rods are vertical at impact, the energy equation becomes

$$\left(2W_t + W_r\right)h = \frac{1}{2}\left(\frac{2W_t}{g}\right)V_i^2 \tag{A-18}$$

where

W_t = Weight of one carriage (lbs)

W_r = Weight of guide rod (lbs)

h = Vertical drop height of guide rod (ft)

V_i = Absolute Velocity of one carriage at impact (fps)

g = Magnitude of acceleration due to gravity (ft/sec^2).

Solving for V_i in Equation A-18

$$V_i = \sqrt{gh\left(2 + \frac{W_r}{W_t}\right)} \ . \tag{A-19}$$

and if $W_r = W_t$

$$V_i = \sqrt{3gh} \ . \tag{A-20}$$